Noisy Coding and Decisions under Uncertainty*

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Abstract

I derive a noisy coding model of decision-making under uncertainty (*NCM*), based on the premise that choice stimuli are mentally encoded by noisy signals, which are optimally combined with pre-existing information about the environment to obtain actionable decision parameters. The parameters emerging from the encoding-decoding process naturally map into prospect theory (*PT*) functionals. The model furthermore makes several novel predictions. The NCM is inherently stochastic, so that no separate error assumptions are needed for data fitting. Noise and model parameters are closely intertwined, resulting in PT violations as well as generating PT-like patterns. Using Bayesian random-parameter models to estimate the model on some prominent datasets originally collected to fit PT, I find substantial support for these predictions. The NCM furthermore consistently outperforms PT in terms of predictive ability. These results contribute to the nascent literature documenting the role of imprecise cognition in economic decisions.

Keywords: risk taking; prospect theory; noisy cognition; ambiguity attitudes

JEL-classification: C93; D03; D80; O12

1 Motivation

Preferences over risk and uncertainty play a central role in economic models. The importance of having accurate descriptive models of decisions under risk has thus long been recognized. As the available evidence on decisions under risk increased, subsequently more sophisticated models of decision making have been proposed over the years (Markowitz, 1952; Kahneman and Tversky, 1979; Loomes and Sugden, 1982; 1986; Gul, 1991; Tversky and Kahneman, 1992; Schmidt, Starmer and Sugden, 2008). Two general

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features stand out in this feedback process between data and models. First, the models were generally created inductively by introducing features that could account for the median or average revealed choice patterns. Second, given the deterministic nature of the models, they need to be augmented by a stochastic component to fit empirical data. Much debate has indeed been generated by the question whether observed behavioural deviations from any given model may be driven by systematic deviations from the choice model itself, or rather by noisy decision patterns pertaining to the stochastic component (Hey and Orme, 1994; Harless and Camerer, 1994; Loomes, 2005).

In this paper, I turn this process on its head by deriving a noisy coding model of decision-making under uncertainty (*NCM*). Actionable choice parameters thereby result from the stochastic mental representation of decision stimuli. Starting from an optimal choice rule entailing expected value maximization, I describe the weights resulting from the optimal combination of noisy signals about choice options with a mental prior indicating their likelihood in the environment. These weights entail nonlinear distortions of both outcomes and likelihoods, which map into the parameters of a popular probability-distortion function frequently used in the PT literature (Goldstein and Einhorn, 1987; Gonzalez and Wu, 1999; Bruhin, Fehr-Duda and Epper, 2010). Both *likelihood-insensitivity*—the stylized fact whereby people are insufficiently sensitive to changes in probabilities in intermediate ranges, while attributing excessive weight to certainty and possibility (Tversky and Wakker, 1995; Fehr-Duda and Epper, 2012)—and decreasing sensitivity towards outcomes—the stylized fact whereby people increasingly discount outcomes as they depart from a reference point—can be traced back to noise in the encoding of the choice stimuli. The NCM can thus be seen as a *generative model for PT* for choices between binary wagers, inasmuch as it naturally results in PT-like decision parameters.[1]

At the same time, however, the NCM makes a number of novel predictions. The model is inherently stochastic, so that no separate assumptions about errors are needed for data fitting. The different model parameters are closely intertwined. For one, this

[1]The derivation of the model starting from first principles means that there are much fewer degrees of freedom than in the inductive derivation of traditional models of choice. This also means, however, that the exact predictions of the model will depend on the details of the decision situation. In this paper, I focus on deriving predictions between binary wagers and sure amounts, which set the stage for the empirical analysis aimed at choice situations following exactly this format. While the model can be generalized to the comparison of wagers with more than 2 non-zero outcomes, such a derivation requires a separate modelling setup building on neural networks allowing for the parallel processing of signals about different branches of a wager, and it is thus left for future research.

close link generates predictions of systematic parameter correlations in estimates of PT parameters. It furthermore results in the prediction that the likelihood and outcome dimensions should interact, thus predicting violations of PT's strict separability precepts. The NCM thus predicts both PT-like functionals, *and* PT violations. The generative nature of the model furthermore results in novel interpretations of the parameters. Given that the prior will plausibly adapt to the stimuli expected in a given situation, the model predicts that parameter estimates gathered in laboratory experiments—where the costs and benefits from taking the wager tend to be equal on average by design—will perform poorly when applied to situations in which costs and benefits are highly asymmetric. Such calibration issues have, for instance, been highlighted in the case of insurance uptake, where typical PT-parameters have been shown to be ill-suited to account for pervasive over-insurance of moderate risks (Sydnor, 2010). The NCM, on the other hand, predicts a pessimistic prior in situations where the potential costs of a wager outweigh its potential benefits, thus providing a new explanation of such insurance choices.

I test these predictions on two large datasets collected to estimate PT parameters under risk—the data of Bruhin et al. (2010), which include three large experiments to identify PT parameters in Switzerland and China, and the data of L'Haridon and Vieider (2019), including 3000 subjects from 30 countries. Estimating individual-level parameters in a Bayesian random-parameter setup, I find the NCM parameters to be closely correlated with the parameters estimated from a PT model plus additive noise[2], illustrating the model's generative nature. I then document systematic correlations between the PT parameters and noise, as well as amongst the PT parameters themselves. While puzzling from the point of view of PT—where the deterministic and stochastic components are generally considered to be orthogonal to each other (a 'white noise' assumption)[3]—these correlations are predicted by the NCM. Finally, I test the NCM

[2]Being deterministic in nature, PT *per se* is silent about errors. Random utility models with additive errors remain the most popular choice in empirical implementations, and they have been applied by both Bruhin et al. (2010) and L'Haridon and Vieider (2019) to the data I use for this comparison. I will thus mainly focus on such a setup when referring to empirical implementations of PT. Such errors could be increased by additional multiplicative terms (Harless and Camerer, 1994; von Gaudecker, van Soest and Wengström, 2011), or contextualized by making them heteroscedastic across utility ranges (Wilcox, 2011). All these approaches have in common that they augment the deterministic model by an independently chosen stochastic mechanism. The NCM, on the other hand, both generates PT-like parameters *and* a stochastic choice model from one and the same underlying process.

[3]Some stochastic models of choice under risk, such the the contextual utility model of Wilcox, 2011, specifically allow for interactions between preferences and decision noise. Nevertheless, the choice of decision model and stochastic model in these setups remains largely independent, and to some extent, arbitrary. We will see below that allowing for such extensions of the error structure within a PT model does not affect the conclusions reached in the analysis.

against PT based on their predictive performance. The NCM significantly outperforms PT in all instances. PT's relatively poor predictive performance can be traced back to the overfitting of certain aspects of the data. The NCM avoids this issue through the integration of noise and decision parameters, which constrains the parameters in ways that give it an edge in terms of both data fit and predictive performance.

The NCM predicts PT's strict separation between the likelihood and outcome dimensions to be violated under risk, with stake changes affecting not only outcome distortions as postulated by PT, but also probability distortions. Such violations are indeed well-documented in the literature (Hogarth and Einhorn, 1990; Fehr-Duda, Bruhin, Epper and Schubert, 2010; Bouchouicha and Vieider, 2017). The NCM predicts that separability should also be violated for decisions under *ambiguity*—a context where the precise decision probabilities are unknown or vague (Ellsberg, 1961)—so that ambiguity attitudes ought to be reflected in both likelihood-distortions and outcome-distortions, rather than purely in the likelihood dimension as predicted by PT. I test these predictions using an original dataset containing choices under both known and vague probabilities, and which is rich enough to allow for the separate assessment of the two dimensions under both risk and ambiguity. Estimating a PT model, I document the typical pattern of likelihood-insensitivity increasing under ambiguity relative to risk (Abdellaoui, Baillon, Placido and Wakker, 2011; Trautmann and van de Kuilen, 2015). At the same time, however, I also find *outcome*-sensitivity to decrease under ambiguity, violating the strict separability precept of PT. These patterns are further strengthened when estimating the NCM, under which they are predicted due to an increase in coding noise when probabilities are vague or unknown. The NCM can thus not only account for typical PT patterns, but it also predicts under what circumstances PT will be violated.

The NCM is generative in nature, inasmuch as it predicts behavioural patterns from the underlying neural decision processes. Behavioural features such as outcome- and likelihood-distortions result from deviations from an optimal choice rule that can be traced back to cognitive constraints imposed by limited cognitive resources. The noisy neural coding of choice stimuli will thus naturally result in discrimination difficulties for values that are observed relatively infrequently, an intuition that is shared with a variety of other models (Robson, 2001; Netzer, 2009; Woodford, 2012; Payzan-LeNestour and Woodford, 2021; Netzer, Robson, Steiner and Kocourek, 2021). Such discrimination difficulties for relatively infrequent stimuli, in turn, will result in systematic biases in those regions. The predictions emerging from the model are thus most apt to explain decisions that are taken quickly and intuitively, rather than being calculated, or decisions that need to be taken in novel environments characterized by high degrees of uncertainty. Discounting signals indicating choice stimuli that are considered unlikely a priori will thus result in a sort of regression to the mean that can explain a number of behavioural regularities (Zhang and Maloney, 2012; Enke and Graeber, 2019). As such, the approach holds the promise of delivering unifying foundation for many decision phenomena that are currently explained by separate models. For instance, Khaw, Li and Woodford (2020) used an analogous setup to explain small-stake risk aversion, which would otherwise appear paradoxical (Rabin, 2000). Noisy coding approaches have furthermore been shown to account for seemingly hyperbolic or present-biased time discounting (Gabaix and Laibson, 2017; Vieider, 2021).

The approach taken here builds on an influential theoretical paradigm in neuroscience, according to which the brain uses probabilistic mechanisms to encode perceptual information about the outside world, and decodes this information by Bayesian updating with a mental prior (Knill and Pouget, 2004; Doya, Ishii, Pouget and Rao, 2007; Vilares and Kording, 2011). Such an approach may indeed be optimal when mental resources are scarce, and need to be distributed between a variety of different tasks. The flip-side of the noisiness in the coding of unlikely events is then that the coding noise may itself optimally adapt to the expected dispersion of the stimuli in the environment—a principle known as *efficient coding* (Barlow et al., 1961; Polania, Woodford and Ruff, 2019; Heng, Woodford and Polania, 2020). Applying an efficient coding setup to the decision model of Khaw et al. (2020), Frydman and Jin (2021) documented systematic differences in revealed risk aversion depending on the dispersion of rewards in the environment. If further augmented by a learning model that allows to acquire the characteristics of the decision environment, this approach thus results in adaptive behaviour that allows to optimally tailor scarce mental resources to the specific circumstances most likely to be encountered in a given environment, thus formalizing some of the fundamental intuitions underlying Herbert Simon's bounded rationality theory (1955; 1959).

This paper proceeds as follows. Section 2 introduces the theoretical framework. Section 3 presents the empirical analysis. I discuss the results in section 4, and section 5 concludes the paper.

2 The Noisy Coding Model

In this section, I lay out the theoretical model in several steps. I start by showing that coding uncertainty as log-likelihood ratios of complementary events constitutes a highly efficient form of representing and updating probabilistic information, thus resulting in an optimal choice rule under uncertainty. I then show how the noisy mental encoding of choices between uncertain prospects yields subjective distortions of the objective quantities in the optimal choice rule. Finally, I use the noisy coding model to derive predictions on decision patterns under risk and ambiguity.

2.1 Noisy coding of uncertainty as log-likelihood ratios

I start by showcasing the efficiency of coding probabilistic information in terms of likelihood ratios. Assume a decision maker (DM) needs to take a decision based on her assessment of the likelihood of an uncertain event e.[4] Assume that the DM does not know the true likelihood of e occurring, but observes a noisy signal, s, about this likelihood. Assume further that the DM knows the probability of the signal conditional on the event, $P[s|e]$, as well as the probability of the signal conditional on the complementary event $\tilde{e}$, $P[s|\tilde{e}] \triangleq 1 - P[s|e]$. The DM can then use the following equation to infer the likelihood of the event, conditional on the signal:

$$\frac{P[e|s]}{P[\tilde{e}|s]} = \frac{P[s|e]}{P[s|\tilde{e}]} \times \frac{\widehat{P}[e]}{\widehat{P}[\tilde{e}]}, \tag{1}$$

where the ratio $\frac{\widehat{P}[e]}{\widehat{P}[\tilde{e}]}$ indicates the prior likelihood ratio (PLR), which incorporates any knowledge the DM may have previously held about the likelihood of event e.[5]

This setup can be used to arrive at an optimal choice rule based on the relative costs and benefits of different actions. Take a wager offering x conditional on event e, but $y < x$ if $\tilde{e}$ obtains instead. Integrating the costs and benefits from taking the wager into

[4]For analytical tractability, I will generally work with discrete events (e.g., a ball extracted from an urn is blue), and associated probabilities $P[e]$. For continuous outcome variables (the stock market index increases by over 2% in a given year; cumulative rainfall during the agricultural planting season falls between 500mm and 700mm), this constitutes a slight abuse of notation for the more accurate $P[a < e < b] = \int_a^b f[e]de$, where f indicates the probability density function.

[5]This follows from an application of Bayes rule, whereby $P[e|s] = \frac{P[s|e]*\widehat{P}[e]}{P[s]}$. The use of likelihood ratios means that $P[s]$ cancels out of the expression.

equation 1, the wager will be preferred to a sure option, c, whenever:

$$\frac{P[e|s]}{P[\tilde{e}|s]} \times \frac{(x-c)}{(c-y)} > 1, \tag{2}$$

where $(x - c)$ indicates the benefit from taking the wager, and $(c - y)$ indicates the cost. This choice rule is optimal, being based on Bayesian updating and expected value maximization.[6] Since any monotonic transformation will leave this choice rule unaltered, we can take the natural logarithm to obtain:

$$ln\left(\frac{P[e|s]}{P[\tilde{e}|s]}\right) > ln\left(\frac{c-y}{x-c}\right). \tag{3}$$

The wager should thus be chosen over the sure option whenever the log-likelihood ratio of event e and its complement $\tilde{e}$ (LLR) exceeds the log cost-benefit ratio ($LCBR$). The use of the logged version of the choice rule in equation 3 instead of its unlogged version is motivated by the observation that this will result in additively separable elements, which are plausible candidates for neural implementation because of their computational tractability (Gold and Shadlen, 2001). Notice, however, that basing the derivations on the choice rule in equation 1 instead does not qualitatively alter the conclusions I draw below. I will return to this issue in due time. While the choice rule can easily be generalized to multiple outcomes (Green, Swets et al., 1966; Gold and Shadlen, 2001), the subsequent mental transformations to be discussed shortly require a different setup building on neural network architectures. In what follows, I will thus focus on tradeoffs between binary wagers and sure outcomes.

The model I will present below is based on the premise that mental processing of uncertain wagers functions much like in the optimal decision rule above. According to an influential theoretical paradigm in neuroscience, the human brain acts as a Bayesian calculation engine, continuously combining noisy signals about the environment with prior beliefs to come up with actionable decision parameters (Knill and Pouget, 2004; Doya et al., 2007; Vilares and Kording, 2011). Even numerically represented quantities, such as a monetary outcome x or an objectively given probability p, will be mentally represented by a noisy signal before entering the choice process. Noise will then arise

[6] This choice rule is used without loss of generality. That is, if the decision-maker instead has a concave utility function defined over lifetime wealth, the choice model can be augmented by such a preference without altering the qualitative conclusions that follow.

especially when assessments of the relevant quantities are made quickly and intuitively, rather than being calculated based on an optimal choice rule (see Khaw et al., 2020, section 1, and Woodford, 2020, for extensive discussions).

2.2 Bayesian mental processing of noisy signals

I now present a step-by-step derivation of subjective choice parameters from the mental encoding-decoding process of choice stimuli. I will show that if the brain implements an optimal choice rule such as derived above, and if the stimuli are mentally encoded with some noise and subsequently decoded using a mental prior, then this process will result in an actionable choice rule into which the encoding noise and the parameters characterizing the mental prior will enter as subjective parameters.

The mental encoding stage and the likelihood

Take a mental signal, r, encoding the characteristics of a given choice problem. This mental signal will take the form of a neural spike-count firing rate, encoding the desirability of the choice stimuli in the brain. The mental signal r thus plays a role that is analogous to that of the signal s used above to illustrate the optimal choice rule. Just like s above, the mental signal will generally be noisy, and the noise itself may carry useful information about the accuracy of the signal.[7] I will assume that there are two such mental signals, r_e and r_o, encoding the desirability of the LLR and the LCBR, respectively.[8] For stimuli that are close together—roughly equal probabilities of e and $\tilde{e}$, or roughly equal costs and benefits—the noisy firing rates will be virtually indistinguishable, resulting in low activations of r_e and r_o. If, however, the LLR or the LCBR deviate significantly from 0, the relative signal will exhibit high spike rates. This phenomenon—known as *discriminability* (Dayan and Abbott, 2001, ch. 3)—will depend both on the difference

[7] More generally, encoding noise may be a function of the cognitive resources available for a given task, in which case we would expect systematic adaptation of the likelihood to the prior (Barlow et al., 1961; Polania et al., 2019; Heng et al., 2020; Frydman and Jin, 2021). This principle, known as *efficient coding*, implies on the one hand that the probability of the mental signal conditonal on the uncertain event, $P[r|e]$, is a known quanity, since the variance of the likelihood will be optimized to detect stimuli in the given choice environment. On the other hand, efficient coding implies that the choice patterns we will derive below may be the consequence of optimal processing under resource constraints rather than constituting decision biases, making optimal use of the information provided by sensory inputs. I will return to this view in the discussion.

[8] The setup with two mental signals corresponds to a minimal setup in which the two choices can be compared. I abstract from explicitly modelling the encoding of other aspects of the decision situation, such as the location on the screen of the risky wager, or the colors associated with winning and losing options, since they are not of central importance to the decision to be taken and do not enter the optimal choice rule derived above.

in the mean spike count rate, and on the noisiness in the distribution.[9]

Stimuli will have compressed mental representations for efficiency reasons, and logarithmic functions are typically used to model such mental representations (Dayan and Abbott, 2001; Dehaene, 2003; Petzschner, Glasauer and Stephan, 2015). I thus assume that r_e and r_o are independently drawn from the following two distributions:

$$r_e \sim \mathcal{N}\left(ln\left(\frac{P[e]}{P[\tilde{e}]}\right), \nu_e^2\right) \quad , \quad r_o \sim \mathcal{N}\left(ln\left(\frac{c-y}{x-c}\right), \nu_o^2\right) \tag{4}$$

where ν_e and ν_o represent the encoding noise of the likelihood ratio and the cost-benefit ratio, conveying information on the uncertainty with which a given stimulus is perceived. Likelihoods of 0 or 1 are assumed to be perceived with certainty. The representation as the logarithm of the stimuli implies that the difference between two stimuli necessary for those stimuli to be reliably discriminated will be proportional to the magnitude of the stimuli themselves. This amounts to a so-called *just noticeable difference* in the stimulus m, Δm, so that $\frac{\Delta m}{m}$ is a constant. The neural encoding process can thus be conceived of as the driving factor behind a behavioural phenomenon that has long been known in psychophysics as the Weber-Fechner law (Fechner, 1860).

The mental decoding stage and the posterior expectations

The information provided by the noisy mental signals r_e and r_o needs to be decoded to be transformed into actionable quantities that can inform the decision process. This is due to the uncertainty in the mental representation of the stimuli, which makes it desirable to combine the encoded signal with prior information about what sort of stimuli are likely to be encountered in the given decision environment. It seems natural to let the mental priors used to this effect follow a normal distribution, since the choice of a conjugate prior distribution will minimize the burden in terms of neural computations:

$$ln\left(\frac{P[e]}{P[\tilde{e}]}\right) \sim \mathcal{N}\left(ln\left(\frac{\widehat{P}[e]}{\widehat{P}[\tilde{e}]}\right), \sigma_e^2\right) \quad , \quad ln\left(\frac{c-y}{x-c}\right) \sim \mathcal{N}\left(ln\left(\frac{\widehat{k}}{\widehat{b}}\right), \sigma_o^2\right) \tag{5}$$

where $ln\left(\frac{P[e]}{P[\tilde{e}]}\right)$ is the mean of the likelihood ratio prior, $ln\left(\frac{\widehat{k}}{\widehat{b}}\right)$ the mean of the cost-benefit ratio prior, and σ_e and σ_o are the associated standard deviations of the priors.

Combining the likelihoods in equation 4 with the priors in equation 5 by Bayesian

[9] Take the neuron pair encoding the cost-benefit ratio in r_o, and designate the neuron as r_o^n and the anti-neuron as r_o^a. Disciminability is then formally defined as $d = \frac{r_o^n - r_o^a}{\tau_o}$, which measures the distance in the means of the activation rates of the two neurons in units of their common standard deviation.

updating, and defining $\xi \triangleq \left(\frac{\widehat{P}[e]}{\widehat{P}[\tilde{e}]}\right)^{1-\gamma}$ and $\zeta \triangleq \left(\frac{\widehat{k}}{\widehat{b}}\right)^{(1-\alpha)}$, we obtain the following posterior expectations of the log-likelihood ratio and log-cost-benefit ratio, conditional on the noisy mental signals (see online appendix S1.1):

$$E\left[ln\left(\frac{P[e]}{P[\tilde{e}]}\right)|r_e\right] = \gamma \times r_e + ln(\xi) \;\;,\;\; E\left[ln\left(\frac{c-y}{x-c}\right)|r_o\right] = \alpha \times r_o + ln(\zeta), \tag{6}$$

where $\gamma \triangleq \frac{\sigma_e^2}{\sigma_e^2+\nu_e^2}$ and $\alpha \triangleq \frac{\sigma_o^2}{\sigma_o^2+\nu_o^2}$ thus constitute the weights assigned to the log-likelihood ratio and the log cost-benefit ratio of the stimulus, relative to the weights assigned to the means of the priors. The relative uncertainty associated with the mental signal and the prior, as captured by $\{\nu_e, \nu_o\}$ and $\{\sigma_e, \sigma_o\}$, will thus determine how much weight will be attributed to the likelihood versus the prior in the posterior distribution, which furnishes the actionable quantities on which the decision will be based.

The actionable choice rule

The posterior means just derived can now be used to inform the decision process. In particular, we must amend the optimal choice rule described in equation 3 by replacing the objective quantities with their mental representations:

$$E\left[ln\left(\frac{P[e]}{P[\tilde{e}]}\right)|r_e\right] > E\left[ln\left(\frac{c-y}{x-c}\right)|r_o\right], \tag{7}$$

indicating that the wager on event e will be accepted whenever the posterior expectation of the log-likelihood ratio exceeds the posterior expectation of the log cost-benefit ratio. Substituting the posterior expectation in equation 6 into the choice rule in equation 7 and solving for the mental signals, we get:

$$\gamma \times r_e - \alpha \times r_o > ln(\delta)^{-1}, \tag{8}$$

where $\delta \triangleq \xi \times \zeta^{-1}$, and where $ln(\delta)^{-1}$ provides the threshold which the weighted difference of mental signals on the left-hand side needs to exceed in order for the wager to be accepted.[10] The threshold parameter δ has a natural interpretation, since it is

[10]Notice that the expectations above are specific to the log-likelihood choice rule in equation 3. If one were to use the unlogged choice rule in equation 1 instead, one needs to take the exponential of the expressions and substitute the parameters on the original scale into the choice rule in equation 7 and take the log to return to the regular scale (see online appendix S1.2). The only difference in the final result will be that the variance of the prior in the likelihood ratio and the cost-benefit ratio would then also contribute to the threshold. Note, however, that this would not be the case if we were to use the median instead of the mean for the decision rule, in which case the two choice rules produce identical

made up by the prior mean of the likelihood ratio, multiplied by the prior mean of the benefit-cost ratio (i.e., the inverse of the cost-benefit ratio)—the more favourable the prior expectation of the likelihood ratio and of the benefit to cost ratio, the more likely the DM will be to accept the wager.

The probabilistic choice rule

To derive the probability with which the wager will be chosen over the sure outcome, we now need to obtain an expression free of the unobservable mental signals. To this end, we obtain the z-score of the weighted difference in mental signals in equation 8 by jointly distributing the two signals exploiting the known distributions in equation 4. Obtaining the z-score, and comparing it to the z-score of the threshold equation 8 (see online appendix S1.1), gives us the following probabilistic choice rule:

$$Pr[(x,e;y) \succ c] = \Phi\left(\frac{ln(\delta) + \gamma \times ln\left(\frac{P[e]}{P[\tilde{e}]}\right) - \alpha \times ln\left(\frac{c-y}{x-c}\right)}{\sqrt{\nu_e^2\gamma^2 + \nu_o^2\alpha^2}}\right), \tag{9}$$

where Φ is the standard normal cumulative distribution function. I briefly discuss two special cases of this equation. If probabilities are perceived objectively and without noise, then γ drops out of the equation, and the standard deviation simplifies to ν_o. If we further let $y \triangleq 0$ and assume that $\zeta = 1$, then $ln(\delta)$ also drops out of the equation and we obtain a special case of the model that corresponds to the noisy perception of outcomes used by Khaw et al. (2020) to explain the Rabin (2000) paradox. For $y \neq 0$, outcome-discriminability is applied to costs and benefits, rather than to single outcomes. If instead we assume a dual scenario under which outcomes are encoded without noise while probabilities are distorted, then α drops out of the equation and the standard deviation simplifies to ν_e.

The setup just derived readily generalizes to a situation where all outcomes are translated into losses. Assume that under event e the DM stands to lose x, or else lose $y < x$ under $\tilde{e}$, and that this scenario is compared to a sure loss of c. The gains and losses are now flipped relatively to the setup discussed above, so that $x - c$ constitutes

results. Körding and Wolpert (2004) show that this is indeed optimal if the absolute value of the error is used as a loss function. Even using the mean, which is optimal for a quadratic loss functions, the *qualitative* predictions remain the same.

the cost from taking the wager, and $c-y$ the potential benefit:

$$E\left[ln\left(\frac{P[e]}{P[\tilde{e}]}\right)|r_e\right] > E\left[ln\left(\frac{x-c}{c-y}\right)|r_o\right], \tag{10}$$

where all derivations follow the same steps as for gains. Such a setup will then naturally result in decreasing sensitivity towards both gains and losses (Woodford, 2012). Khaw et al. (2020) furthermore show that loss aversion can arise from differences in the prior for gains and losses—a finding that carries over to the setting I use in this paper, and which I thus do not discuss further.

2.3 The NCM generates PT-like functionals for binary wagers

I next discuss the implications of the model I have derived above, and show that for 2-outcome wagers the setup results in a stochastic version of prospect theory. Abstracting momentarily from decision noise, we can write the point of indifference underlying the choice probability in equation 9 as follows:

$$ln\left(\frac{c-y}{x-c}\right) = \alpha^{-1}\left[ln(\delta) + \gamma \times ln\left(\frac{P[e]}{P[\tilde{e}]}\right)\right]. \tag{11}$$

This represents the point of indifference between $(x, e; y)$ and c, where the likelihood dimension on the right-hand side—subjectively transformed by the mental representation parameters α, γ, and δ—is traded off against the outcome dimension on the left-hand side. Setting $P[\tilde{e}] = 1 - P[e]$, the expression on the left can be transformed as follows:

$$ln\left(\frac{c-y}{x-c}\right) = ln\left(\frac{\frac{c-y}{x-y}}{1-\frac{c-y}{x-y}}\right) \triangleq ln\left(\frac{\pi(P[e])}{1-\pi(P[e])}\right), \tag{12}$$

where $\pi(P[e])$ is the solution of the equation $c = \pi(P[e])x + (1-\pi(P[e]))y$, constituting a dual-EU representation of the choice problem (Yaari, 1987), for the decision weight $\pi(P[e])$. The right-hand side in equation 12 can thus be interpreted as the log of the ratio of the decision weights assigned to the high and to the low outcome in the wager. Substituting equation 12 into equation 11, and solving for $\pi(P[e])$, we obtain:

$$\pi(P[e]) = \frac{\delta^{1/\alpha}(P[e])^{\gamma/\alpha}}{\delta^{1/\alpha}\left(P[e]\right)^{\gamma/\alpha} + (1-P[e])^{\gamma/\alpha}}. \tag{13}$$

For $\alpha = 1$, this expression reduces to a probability-distortion function commonly used in the decision-making literature (Goldstein and Einhorn, 1987; Gonzalez and Wu, 1999; Bruhin et al., 2010). The general case of $\alpha \leq 1$ allows for outcome distortions in addition to probability distortions, and shows how probability weighting emerges from the tension between the two. The parameter γ mostly governs the slope of the function, capturing likelihood-sensitivity. The fact that γ decreases in the noisiness of the coding process receives support from findings showing that probabilistic sensitivity increases with cognitive ability (Choi, Kim, Lee, Lee et al., 2021). The parameter δ determines mostly the elevation of the function, and thus has an interpretation of optimism when used as a weighting of ranked gains, and of pessimism when applied to ranked losses.[11]

The mapping just presented shows how PT-like parameters naturally emerge from noisy mental representations of choice stimuli and their mental decoding by a prior distribution, indicating the likelihood of different stimuli in a given decision environment. A difference from PT is that outcome-distortions are defined over the costs and benefits associated with different events, rather than over single outcomes. Furthermore, the parameters α and γ are naturally restricted to fall between 0 and 1, whereas under PT they are only restricted to be positive. I will discuss the implications of these restrictions extensively in the empirical analysis below. Most importantly, however, the derivation of the PT-like parameters from the native NCM parameters results in a new interpretation of the PT parameters themselves. This is indeed the sense in which the NCM is a *generative* model for PT—it presents a causal account of how the PT parameters come about from a probabilistic mental representation of uncertain choice stimuli.

We can further derive substantive predictions about behaviour from the intuitions emerging from the model. Under the NCM, the parameter $\delta \triangleq \xi \times \zeta^{-1}$ has a natural interpretation as the mean of the prior. Assume that $\zeta = 1$, i.e. that the prior indicates costs and benefits that are equal on average (as realistic assumption for typical experimental setups). The constituent part of its likelihood-specific component ξ, $\psi \triangleq \widehat{P}[e]$, can then be shown to coincide with the fixed point of the probability-distortion function, i.e. the point where the function crosses the 45° line. Substituting ψ into the decision

[11]Looking at this equation in isolation it may appear that for $\delta = 1$ the two parameters α and γ are not separately identified. This is, however, not the case, since α and γ also enter into the noise term. I will discuss the empirical identification of the model parameters in section 3.1.

weight we obtain:

$$ln\left(\frac{\pi(\psi)}{1-\pi(\psi)}\right) = \gamma \times ln\left(\frac{\psi}{1-\psi}\right) + (1-\gamma) \times ln\left(\frac{\psi}{1-\psi}\right) = ln\left(\frac{\psi}{1-\psi}\right), \tag{14}$$

from which follows $\pi(\psi) = \psi$, and by extension, $\pi(P[e]) = P[e]$. In other words, the expectation of the mental prior coincides with the fixed point of the probability-distortion function, at which probabilities are perceived without subjective distortions.

Take now the general case in which $\zeta \neq 1$. The benefit-cost prior ζ^{-1} suggests that optimism ought to be particularly high in situations that present a large potential benefit with a small liklihood, as may be the case for lottery play. The inverse happens for large potential losses, where the parameter captures pessimism. This, in turn, ought to increase the likelihood of insurance uptake due to the large downside of the wager relative to the cost of the insurance. Such a pessimistic prior may then explain calibration issues that have been observed when PT parameters estimated in lab experiments are used to explain insurance decisions (Sydnor, 2010), which is why subsequent approaches have used a combination of probability distortions and loss aversion to explain such choices (Barseghyan, Molinari, O'Donoghue and Teitelbaum, 2013). That is, the prior for lab experiments—where costs and benefits tend to be equal on average—should not be expected to be the same as in an insurance context, where the large potential downside of the loss ought to result in increased levels of pessimism.

The setup above can furthermor be immediately applied to decision-making under risk and ambiguity. The case of risk obtains directly by setting $P[e] = p$, with p an objectively known probability. The case of ambiguity obtains when subjects are asked to bet on Ellsberg-urns with unknown colour proportions (Ellsberg, 1961; Abdellaoui et al., 2011). Given that unknown odds are more difficult to encode than known odds (Petzschner et al., 2015), we should expect coding noise to increase as the information about the probabilities involved becomes more vague.[12] This, in turn, yields the prediction that $\gamma_a < \gamma_r$, where the subscripts a and r stand for 'ambiguity' and 'risk' respectively—a phenomenon known as *ambiguity-insensitivity*, and which has been found

[12]In keeping with the rest of the paper, I limit my discussion to the case of Ellsberg urns with unknown color proportions. Findings may well deviate from the ones described here in other contexts and for different sources of uncertainty, since it is well-known that ambiguity attitudes over natural sources of uncertainty may vary depending on a DM's knowledge of the specific context or the DM's perceived competence in a given task (Heath and Tversky, 1991; Fox and Tversky, 1995; Abdellaoui et al., 2011). A good descriptive fit to choices under genuine uncertainty would thus require augmenting the decision model here presented with a learning model for the prior, which is beyond the scope of this paper.

to be near-universal (Abdellaoui et al., 2011; Dimmock, Kouwenberg and Wakker, 2015; Trautmann and van de Kuilen, 2015; L'Haridon, Vieider, Aycinena, Bandur, Belianin, Cingl, Kothiyal and Martinsson, 2018). This prediction is further supported by the finding that time pressure, which presumably serves to augment coding noise, increases ambiguity-insensitivity (Baillon, Huang, Selim and Wakker, 2018). Another implication of increased encoding noise under ambiguity is that we would simultaneously expect an uptick in decision noise under ambiguity—a prediction that is born out by the increase in noisiness under ambiguity relative to risk documented by L'Haridon et al. (2018).

The prior mean, δ may further be affected by changes in discriminability across decision situations. Since $\delta_a = \left(\frac{\psi_a}{1-\psi_a}\right)^{1-\gamma_a}$, the prior is directly impacted by the discriminability parameter γ_a unless $\psi_a = 0.5$. Furthermore, the fixed point of the function need not be the same as under risk, i.e. generally $\psi_a \neq \psi_r$. Ambiguity aversion—a dislike of unknown probabilities unrelated to their log-likelihood ratio—would then enter the model as a pessimistic prior, resulting in $\psi_a < \psi_r$. This may constitute a plausible assumption in conditions where the experimenter may be in a position to deceive subjects, as is the case when a colour choice is not allowed in urn choice problems, or more generally because of the impression generated by the inherent setup of the Ellsberg experiment that the experimenter is expressly withholding relevant information (Frisch and Baron, 1988; Fox and Tversky, 1995).

2.4 Stochasticity and violations of probability-outcome separability

It is now at the time to circle back to the decision noise parameter. Other than under typical stochastic implementations of PT, where the decision model and the noise model are combined in an *ad hoc* fashion, decision noise naturally emerges from the same mental encoding-decoding process as the other model parameters. Although conceptually distinct, encoding noise ν_e and ν_o cannot be distinguished in practice from observed choices unless one is willing to make strong additional assumptions about the underlying process. This follows from the joint distribution of r_e and r_o (see equation 18 in the online appendix), showing that this inseparability is a feature of the underlying mental coding process. I will thus henceforth write $\nu_e = \nu_o \triangleq \nu$, which results in a simplified decision error $\omega \triangleq \nu \times \sqrt{(\alpha^2 + \gamma^2)}$. Assuming that $\zeta = 1$—a realistic assumption for the experimental data I will examine below—the NCM has the same number of free parameters as a PT model plus additive noise.[13]

The decision noise parameter ω holds the key to understanding the NCM. PT is most often estimated in conjunction with an additive decision noise variable that is assumed to be orthogonal to the deterministic decision model (see section 3.1 for a short formal exposition of a typical PT setup). In the NCM, on the other hand, the noisiness of the choice process is intimately and inextricably linked to the other parameters of the model. Outcome-discriminability α and likelihood-discriminability γ directly enter the definition of decision noise ω, and through them their component parts σ_o and σ_p (on top of ν). That implies that the noisiness of the decision process will be impacted both by the uncertainty connected to the stimulus encoding, and the dispersion of the mental priors. Model parameters and decision noise thus move together, resulting in an inherently stochastic model. An exception to this rule concerns the mean of the prior. While optimism δ is affected through γ, given its definition $\delta = \left(\frac{\psi}{1-\psi}\right)^{1-\gamma}$, the native NCM parameter ψ making up the mean of the prior distribution remains unaffected. This will have important interpretative consequences, to which I will return in the discussion.

The inherent stochasticity of the NCM has a number of substantive implications for the decision-making patterns we would expect under risk and ambiguity. For one, parameters estimated in a PT setup, which neglects these intricate interrelations, should be correlated in a systematic way. This concerns primarily correlations between the noise parameter and likelihood- and outcome-sensitivity, although there are likely to be also knock-on effects on other parameters. The neglect of these systematic correlations, in turn, may result in inferior predictive ability of the model. As a consequence, different model parameters estimated under PT will jointly be affected by the same observable characteristics of the decision maker or the environment, with model parameters often moving in opposite directions in terms of revealed risk attitudes with the observable characteristics of DMs, making an analysis of the correlates of risk attitudes based on PT difficult to interpret.

A further consequence of the inextricability of ν_e and ν_o is that no strict separability

[13] Focusing on prospects over gains only, the NCM has the native parameters ν, indicating encoding noise; ψ, indicating the fixed point of the probability-distortion function; and the two standard deviations of the likelihood and outcome priors, σ_e and σ_o. This corresponds to PT's 3 deterministic parameters $\widehat{\alpha}$ (utility curvature), $\widehat{\gamma}$ (likelihood-sensitivity), and $\widehat{\delta}$ (optimism), plus a noise parameter, call it $\widehat{\omega}$, where the 'hat' on the parameters serves to distinguish the native PT parameters from the equivalent NCM-derived parameters discussed in the text. These parameters are exactly replicated for losses, under the assumption that the error term under PT is allowed to be heteroscedastic across choice domains.

between the likelihood and outcome dimensions seems warranted, in opposition to what is assumed in PT. I illustrate the consequences with two examples. In first instance, we may expect that coding noise increases in the range of the stakes. This sort of adaptation may indeed be optimal in a context of scarce resources. Frydman and Jin (2021) present a model of *efficient coding*, in which the parameter of the likelihood function adapts to the prior exactly in this way. They furthermore provide experimental evidence that when outcome ranges increase, so does the noisiness in the encoding process. Such an increase in noisiness should then immediately be reflected in lower outcome-discriminability, resulting in apparent patterns of increasing relative risk aversion (Holt and Laury, 2002). Given the impossibility of separating the different dimensions of encoding noise, however, it will also result in apparent changes in likelihood-discriminability, thus impacting the probability dimension. Under PT, such effects will take the form of violations of probability-outcome separability, whereby changes in stakes ought to be reflected purely in utility curvature and leave probability-distortions unaffected. Such violations are well-documented in the literature (Hogarth and Einhorn, 1990; Fehr-Duda et al., 2010; Bouchouicha and Vieider, 2017).

The flip-side of this issue is observed when moving from risk to ambiguity. Given the increase in encoding noise of likelihood ratios we expect under ambiguity, the NCM predicts a lowered level of likelihood-discriminability. The latter, in turn, is expected to go hand-in-hand with a lowering of outcome-discriminability. While remaining undocumented to date, such a pattern would contradict PT, according to which ambiguity attitudes ought to be reflected purely in probability weighting (Wakker, 2010; Abdellaoui et al., 2011; Dimmock et al., 2015).[14] Notice that the theoretical limiting case is one whereby coding noise for outcomes and probabilities is completely independent, such that $\nu_e \perp \nu_o$, in which case we would indeed expect outcome-discriminability to be unaffected by properties pertaining to the probability dimension, as predicted by PT. That case, however, appears conceptually unlikely due to the reasons laid out above.

[14] It is, of course, also in contradiction to models that capture ambiguity attitudes purely through utility curvature, such as the one proposed by Klibanoff, Marinacci and Mukerji (2005).

3 Empirical evidence

In this section, I use several datasets to test the predictions and performance of the NCM. For space reasons, only essential details are provided in the main text, while additional details and results are provided in the online appendix.

3.1 Preliminaries to empirical estimations

I use Bayesian random parameter models to estimate the model parameters (Gelman and Hill, 2006; Gelman, Carlin, Stern, Dunson, Vehtari and Rubin, 2014b). This allows me to obtain individual-level estimates of all the parameters, while estimating the full variance-covariance matrix of the model parameters. This method is geared towards maximizing the *predictive* power of the model for new data, rather than towards optimizing its fit to existing data, to contain issues arising from overfitting. I conduct the analysis using Stan (Carpenter, Gelman, Hoffman, Lee, Goodrich, Betancourt, Brubaker, Guo, Li and Riddell, 2017), and I test model performance by means of leave-one-out cross-validation (*LOO*; Gelman, Hwang and Vehtari, 2014a; Vehtari, Gelman and Gabry, 2017). Models are thus compared on their out-of-sample predictive ability, rather than based on their fit to a given dataset.[15] Correlations discussed in the text refer to Spearman rank correlations on the estimated parameters. Any p-values reported are always two-sided. Figures may cut some outliers for better visual display.

I am interested in two sets of parameters. The first set consists in the NCM parameters, which consist of two sub-groups: 1) the native NCM parameters ν, ψ, σ_e, and σ_o; and 2) the NCM-derived PT parameters α, γ, δ, and ω. I refer to γ as *likelihood-discriminability* and to α as *outcome-discriminability*. The second set of parameters I am interested in are the native PT parameters resulting from a typical PT setup plus additive noise—the most frequently used stochastic implementation of PT, and the one that is used in the both papers from which I take the data. Under PT, a prospect $(x, p; y)$ will be chosen over a sure outcome c whenever

$$c < u^{-1}[w(p)u(x) + (1 - w(p))u(y)] + \epsilon, \tag{15}$$

[15] The results obtained with the LOO criterion are similar to, but more stable than, those obtained using the Watanabe-Akaike information criterion (WAIC; Watanabe and Opper, 2010), which is a Bayesian generalization of the deviance information criterion. All model comparisons reported are unaffected by using the WAIC instead.

where $w(p)$ is a probability-distortion function mapping probabilities into decision weights, and $u(.)$ is a utility function mapping outcomes into utilities. This model is applicable to both gains and losses. To allow for data fitting, an additive error term $\epsilon \sim \mathcal{N}(0, \widehat{\omega}^2)$ is appended to the deterministic part of the model. This random error is customarily interpreted to take the form of 'white noise', i.e. implicitly assumed to be orthogonal to the deterministic part of the model.[16] To ensure comparability with the NCM, I will use power utility throughout, so that $u(x) = x^{\widehat{\alpha}}$. I will use the Goldstein and Einhorn (1987) probability-distortion function for the same reason, with parameters $\widehat{\gamma}$ and $\widehat{\delta}$, where the 'hat' serves to distinguish the PT parameters from the equivalent NCM-generated parameters.[17] I will refer to $\widehat{\gamma}$ as *likelihood-sensitivity*, and to $\widehat{\alpha}$ as *outcome-sensitivity*, to distinguish them from the equivalent NCM-derived parameters.

Under risk, the NCM parameters are identified whenever the PT parameters are, thus requiring sufficient variation over both probabilities and outcomes. A crucial element for both models is the inclusion of non-zero lower outcomes, without which the model parameters are only unique up to a power. All data I use fulfil this criterion. Ambiguity attitudes are trickier to identify, given how the NCM makes different predictions from PT. This means that datasets collected to fit PT models of ambiguity attitudes are generally not suited to identify NCM parameters. I will thus limit my analysis to only one original dataset, which includes the required nonzero lower outcomes needed to identify outcome-distortions under ambiguity.

Noise deserves some extra attention in this context, given its centrality to the NCM setup. Khaw et al. (2020) identify noisiness by repeating identical choices multiple times. Noise, however, can also be identified in contexts where different choices are similar, though not identical, since such choices will result in inconsistencies and stochastic dominance violations across choice lists in regions where stimuli cannot be discriminated. Experiments imposing or nudging subjects towards consistency within choice lists may somewhat underestimate noise. Notice, however, that this would bias the tests I con-

[16]Many implementations allow for some sort of heteroscedasticity of the error term. For instance, both Bruhin et al. (2010) and L'Haridon and Vieider (2019) allow errors to be heteroscedastic across gains and losses, as well as proportional to the outcome range in the given prospect. Contextualizing the error by making it proportional to the *utility* range, as proposed by Wilcox (2011), does not affect any of the conclusions I draw below.

[17]Prelec (1998) presents an popular alternative 2-parameter weighting function that has been used inter alia by L'Haridon and Vieider (2019). Notice, however, that the two weighting functions provide a very similar fit except for prospects with extremely small or extremely large probabilities, which do not occur in the data I analyze.

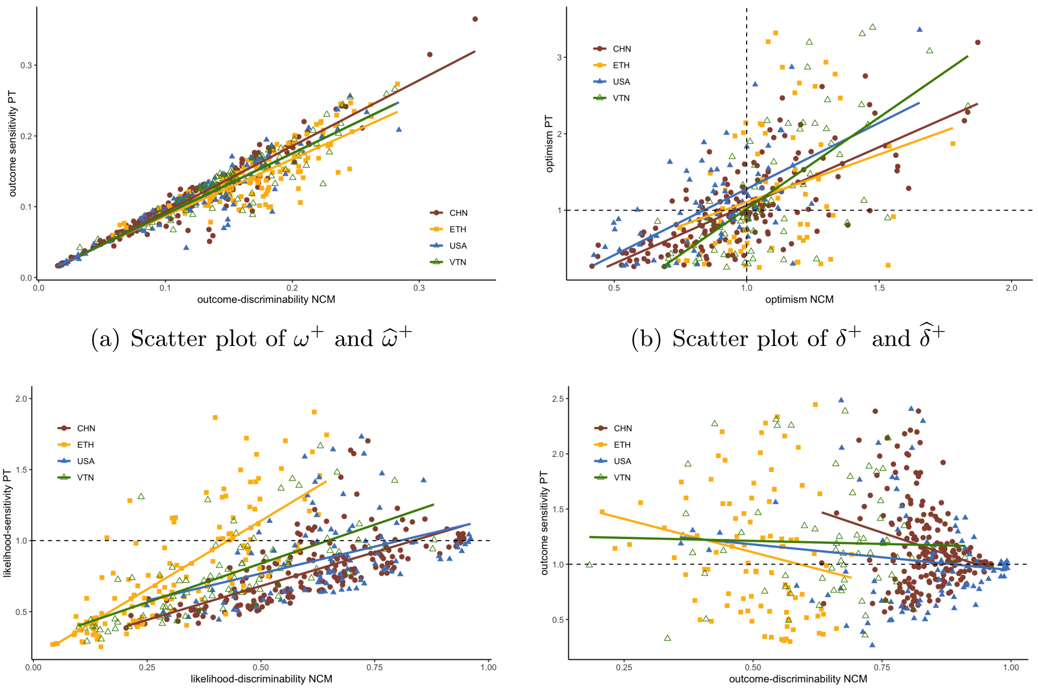
duct against the NCM, so that my tests on data collected to fit PT can be considered conservative.

3.2 The NCM generates PT patterns

In now illustrate some stylized patterns for two-outcome prospects based on the data of Bruhin et al. (2010) and L'Haridon and Vieider (2019). The details of the econometric approach, as well as a host of additional results and graphs for the two papers, can be found in the online appendix. I start by illustrating the generative nature of the NCM. To this end, I use a few selected countries from the data of L'Haridon and Vieider (2019) to illustrate the correlations between the NCM-derived PT parameters, α, γ, δ, and ω and the corresponding native PT parameters, $\widehat{\alpha}$, $\widehat{\gamma}$, $\widehat{\delta}$, and $\widehat{\omega}$. The four countries selected are representative of the overall trends in the data, but allow for a more nuanced visualization. Results for other data—e.g. for other countries, for losses, or for the Bruhin et al. (2010) data— are similar and can be found in the online appendixes.

The correlations are displayed in figure 1. The correlation between decision noise in the two models, shown in panel 1(a), is extremely tight ($\rho = 0.947, p < 0.001$), as one might expect given that both are identified from the noisiness of choices. Given that ω contains in itself the model parameters α and γ, whereas $\widehat{\omega}$ is implemented as an independent dimension, we would however expect larger differences to emerge for those other parameters. These differences start surfacing in panel 1(b), showing the correlation of the optimism parameter emerging from the two models ($\rho = 0.711, p < 0.001$). The distribution of the parameter under the NCM is much narrower when compared to the equivalent parameter in the PT model (IQR of 1.25 for $\widehat{\delta}$ versus 0.336 for δ), and is a manifestation of the shrinking effect of the $1 - \gamma$ power applied to the prior mean in the NCM. The parameter correlations decrease further once we move to likelihood distortions, shown in panel 1(c) ($\rho = 0.662, p < 0.001$). Notice also that values of $\widehat{\gamma} > 1$ do not generally map into values of γ close to 1, but rather into intermediate values.

Once we move to outcome-distortions, shown in panel 1(d), we no longer find any systematic correlation between the PT and NCM parameters ($\rho = -0.031, p = 0.522$). This absence of correlation is based on two main factors. The first is the different definition of outcome-distortions in the two models. Whereas PT applies outcome distortions to individual outcomes, the NCM results in distortions applied to the costs and benefits



(a) Scatter plot of ω^+ and $\widehat{\omega}^+$

(b) Scatter plot of δ^+ and $\widehat{\delta}^+$

(c) Scatter plot of γ^+ and $\widehat{\gamma}^+$

(d) Scatter plot of α and $\widehat{\alpha}^+$

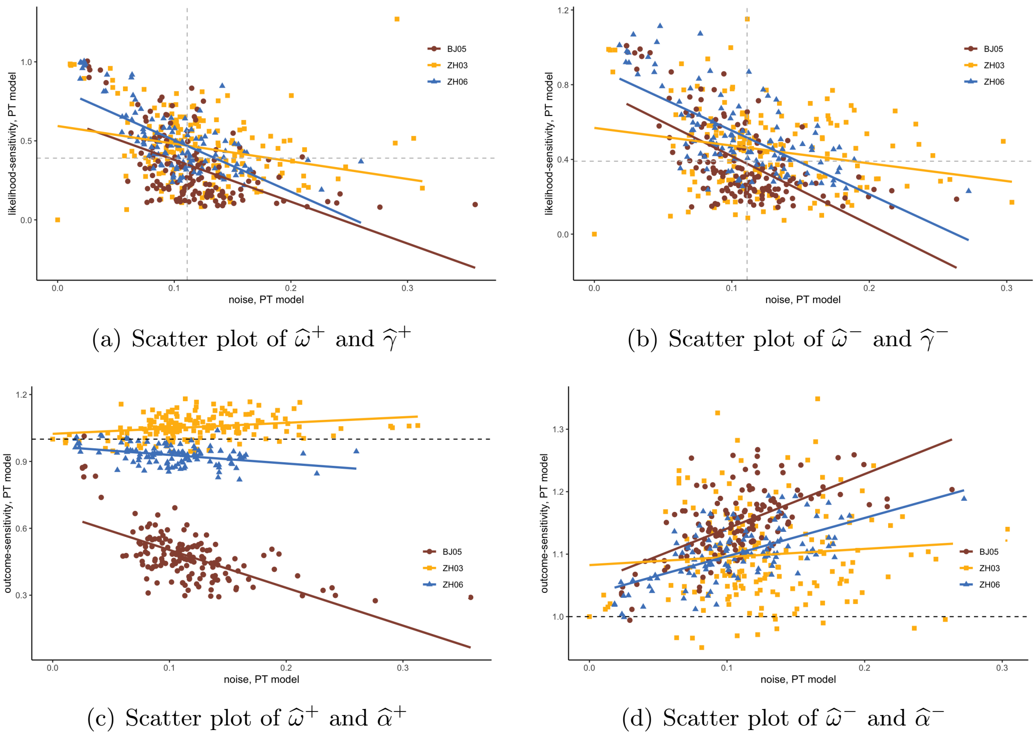
Figure 1: NCM-generated parameters and PT parameters for L'Haridon and Vieider (2019)

of a wager. The second reason concerns the coexistence of values of $\widehat{\alpha}$ larger and smaller than 1 under PT. We will see shortly that both are associated with noise. Indeed, values of the parameters falling relatively close to 1, which tend to be associated with low noise levels (see below), are very similar across the two models.

3.3 PT parameters show systematic inter-correlations

We have seen above that the NCM predicts systematic correlations in the PT parameters it generates, due to encoding noise entering the definition of both likelihood-discriminability and outcome-discriminability. The latter, in turn, my impact optimism/pessimism though their shrinking function for those parameters. To the extent that PT parameters are indeed generated by noisy coding, we should thus expect to find systematic correlations between PT parameters. Figure 2 shows correlations between decision noise $\widehat{\omega}$ and likelihood-sensitivity $\widehat{\gamma}$ and outcome-sensitivity $\widehat{\alpha}$ for both gains and losses in the data of Bruhin et al. (2010) (the patterns in the data of L'Haridon and Vieider (2019) are very similar, and are illustrated in online appendix S2.3).

Panel 2(a) shows a scatter plot of the PT noise parameter, $\widehat{\omega}^+$, against the likelihood-



(a) Scatter plot of $\widehat{\omega}^+$ and $\widehat{\gamma}^+$

(b) Scatter plot of $\widehat{\omega}^-$ and $\widehat{\gamma}^-$

(c) Scatter plot of $\widehat{\omega}^+$ and $\widehat{\alpha}^+$

(d) Scatter plot of $\widehat{\omega}^-$ and $\widehat{\alpha}^-$

Figure 2: Scatter plot of PT parameters
The parameters have been obtained from the estimation of a PT model plus additive noise. The different colours and shapes represent the 3 experiments in Bruhin et al. (2010): ZH03 stands for Zurich 03; ZH6 for Zurich 06; and BJ05 for Beijing 05. The dashed lines indicate the median parameter values in panels 2(a) and 2(b), while indicating the salient point of 1 in panels 2(c) and 2(d).

sensitivity parameter, $\widehat{\gamma}^+$, for gains. The two parameters show a strong negative correlation ($\rho = -0.422, p < 0.001$), which is present in each of the 3 experiments. The results for losses, shown in panel 2(b), are very similar ($\rho = -0.389, p < 0.001$). For both gains and losses, a small group stands out that has virtually no noise and likelihood-sensitivity arbitrarily close to 1. These are the expected value maximizers detected in the mixture model of Bruhin et al. (2010), who most likely based their responses on precise calculation of expected values rather than on quick and approximate judgments.

Panel 2(c) shows the correlations between the PT noise parameter, $\widehat{\omega}^+$, and the outcome sensitivity parameter, $\widehat{\alpha}^+$, for gains. Once again we witness systematic correlations, although the correlation is negative in the Beijing 05 data ($\rho = -0.566, p < 0.001$), as well as in the Zurich 06 data ($\rho = -0.357, p < 0.001$), but positive in the Zurich 03 data ($\rho = 0.279, p < 0.001$). For utility, noise can thus be correlated with either excess sensitivity or insensitivity under PT. This intuition is further confirmed for losses, shown in panel 2(d). Here we witness a positive correlation in the aggregate data ($\rho = 0.258, p < 0.001$), as well as in the three individual experiments (ZH03: $\rho = 0.07, p = 0.38$; BJ05: $\rho = 0.612, p < 0.001$; ZH06: $\rho = 0.558, p < 0.001$). This is driven by concave utility for losses in all three experiments. Such patterns could, for instance, arise if noisy choice data are overfit. I will discuss this issue at some length further below in the context of the NCM.

The correlations just shown further have knock-on effects on correlations between the deterministic model parameters. In particular, likelihood-sensitivity $\widehat{\gamma}$ and outcome-sensitivity $\widehat{\alpha}$ will tend to be either positively or negatively correlated, depending on whether the correlation of $\widehat{\omega}$ with $\widehat{\alpha}$ is negative or positive. For instance, likelihood sensitivity and outcome-sensitivity for gains are positively correlated in the Beijing 05 data ($\rho = 0.463, p < 0.001$), as well as in the Zurich 06 data ($\rho = 0.753, p < 0.001$). They are, however, negatively correlated in the Zurich 03 data ($\rho = -0.254, p < 0.001$), given the positive correlation between decision noise and outcome-sensitivity. In terms of optimism δ^+ and pessimism δ^-, larger values of gamma tend to compress the distributions towards one (see online appendices for graphs and tests).

3.4 The NCM beats PT in predictive accuracy

A natural next question concerns the predictive performance of the two models. Table 1 reports test statistics for Bruhin et al. (2010), separately for their three experiments, and further separated for gains and losses. The NCM outperforms PT in all 6 test cases, and the difference in performance is large. The good performance of the NCM using the Bruhin et al. (2010) data is in no way exceptional. Table S2 in the online appendix provides tests of the predictive ability of the NCM versus PT using the data of L'Haridon and Vieider (2019). I conduct the tests country by country, and separately for gains and losses. Conditional on the design, this gives me 60 independent tests. Once again, the NCM easily outperforms PT in all 60 cases, with all tests statistics two orders of magnitude larger than the associated standard errors.

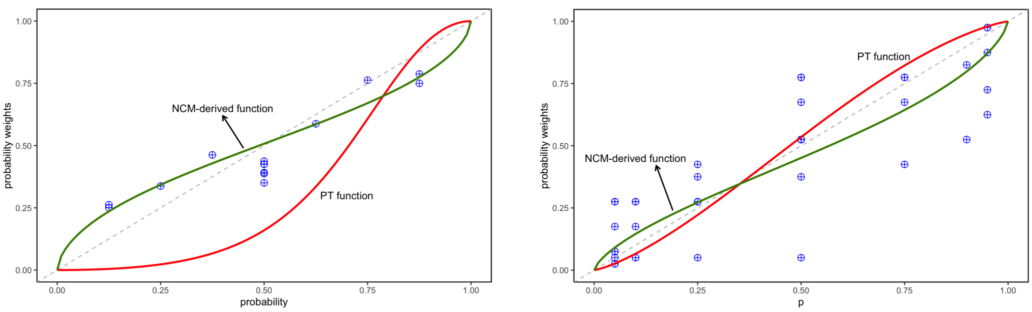
The superior predictive performance of the NCM as compared to PT may appear puzzling in the light of the fact that the NCM does not allow for discriminability parameters larger than 1, i.e $\alpha \leq 1$ and $\gamma \leq 1$. This is purely due to the fact that the NCM avoids the overfitting of noisy data, which remain a problem in the PT setup notwithstanding the hierarchical model geared towards discounting noisy outliers. This

Table 1: Evidence in favour of NCM over PT plus additive noise

	gains	losses
Zurich 03	−14084.9	−13942.6
(SE)	(46.3)	(41.7)
Zurich 06	−7824.2	−7794.7
(SE)	(33.0)	(33.1)
Beijing 05	−4768.5	−4778.0
(SE)	(31.5)	(30.8)

Test results refer to LEPD differences (log estimated predicted density differences), and indicate the comparative performance of the two models (Gelman et al., 2014a; Vehtari et al., 2017). Negative differences constitute evidence in favour of the NCM over PT. Tests using WAIC yield virtually identical results, and are thus not reported for parsimony.

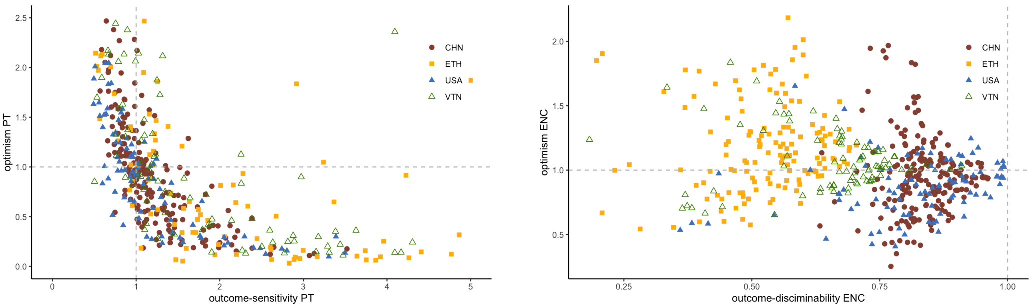
fact is best illustrated with an example. Take subject 319 from L'Haridon and Vieider (2019). His PT parameters indicate extreme likelihood-sensitivity, $\widehat{\gamma} = 1.91$, joint to very low levels of optimism, at $\widehat{\delta} = 0.16$. The noisiness of his decisions is relatively high, at $\widehat{\omega} = 0.19$. Panel 3(a) shows the fit of this probability-distortion function to the raw data points, which are obtained under the assumption of linear utility. The fit appears to be terrible. At the same time, the fit of the function based on the NCM-derived PT parameters appears much more reasonable. The example from the data of Bruhin et al. (2010) shown in panel 3(b) reveals similar issues, albeit muted by the multiplicity of observations.



(a) Example from L'Haridon and Vieider (2019) (b) Example from Bruhin et al. (2010)

Figure 3: Examples of probability-distortion fit for PT vs the NCM
Fit of estimates probability-distortion functions derived from PT vs the NCM. Non-parametric data points are shown in blue, and are derived under the assumption that $u(x) = x$. Decision weights $\pi(p) = \frac{c-y}{x-y}$ are then plotted on the y-axis against the probability on the x-axis.

The underlying reason for the poor fit of the PT function can be traced back to the other parameters of the model, and particularly tends to go hand-in-hand with extreme values of outcome-sensitivity (the value for the subject in panel 3(a) is $\widehat{\alpha} = 3.74$; outcome-sensitivity for the subject in panel 3(b) is $\widehat{\alpha} = 1.05$). The outcome-discriminability parameter under the NCM tends to have much more reasonable values due to the constraints emerging from the definition of the parameter. Taking into account the role of α in contributing to noise, this results in much better predictive performance, as well as improving the fit of the probability-distortion function to the data. Section S2.5 in the online appendix shows more example of individual-level data fit.



(a) Scatter plot of $\widehat{\alpha}^+$ and $\widehat{\delta}^+$

(b) Scatter plot of α and δ

Figure 4: Scatter plot of outcome-distortion and optimism parameters for L'Haridon and Vieider (2019)

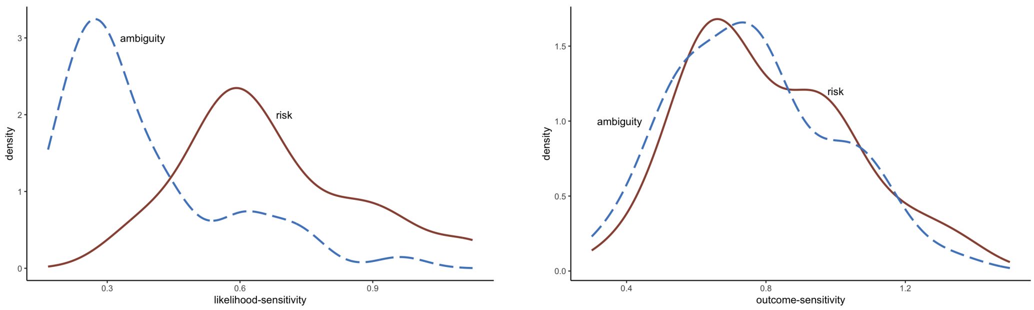
The pattern in the examples just shown is far from exceptional. Indeed, the poor fit of PT, and its even poorer performance in terms of predictive accuracy, is largely driven by the poor separability of some of its parameters, and their assumed orthogonality to noise. One particularly problematic dimension is the correlation between outcome-sensitivity $\widehat{\alpha}$ and optimism $\widehat{\delta}$ under PT. This correlation is shown in panel 4(a) of figure 4. It can be seen to be L-shaped, with large values of outcome-sensitivity corresponding to low levels of optimism, and vice versa. Since values of outcome-sensitivity larger than 1 are further associated with noise, this pattern points at poor separability of the parameters in a traditional PT setup (Zeisberger, Vrecko and Langer, 2012). The equivalent parameters α and δ in the NCM shown in panel 4(b) appear to be orthogonal to each other.

3.5 Violations of probability-outcome separability under ambiguity

From the discussion above it follows immediately that the NCM predicts ambiguity-insensitivity, as well as allowing for ambiguity aversion. These patterns are well-documented in the literature, and can be fit well by PT. The NCM, however, also makes additional predictions. For one, we would expect coding noise, which I denote by ν_a to emphasize that it is specific to ambiguity, to be negatively correlated with ambiguity-insensitivity. In addition, we would expect coding noise to be substantially larger than under risk, so that $\nu_a > \nu_r$, where the subscript r indicates known probabilities.

Published data on ambiguity attitudes tend to be somewhat limited in that they have been collected explicitly to elicit parameters under the assumption that PT holds (or else for nonparametric analysis). That is, they typically lack the sort of stimuli that would allow one to assess whether $\widehat{\alpha}$ may be affected by ambiguity in addition to $\widehat{\gamma}$, since utility curvature for risk and ambiguity is assumed to be the same under PT due to the strict separation of the likelihood and outcome dimensions inherent in the model (Wakker, 2010; Abdellaoui et al., 2011; Dimmock et al., 2015). This strict separation does not hold in the NCM due to the effect of encoding noise on both γ and α, resulting in a prediction that both likelihood-discriminability *and* outcome-discriminability should be affected by ambiguity.

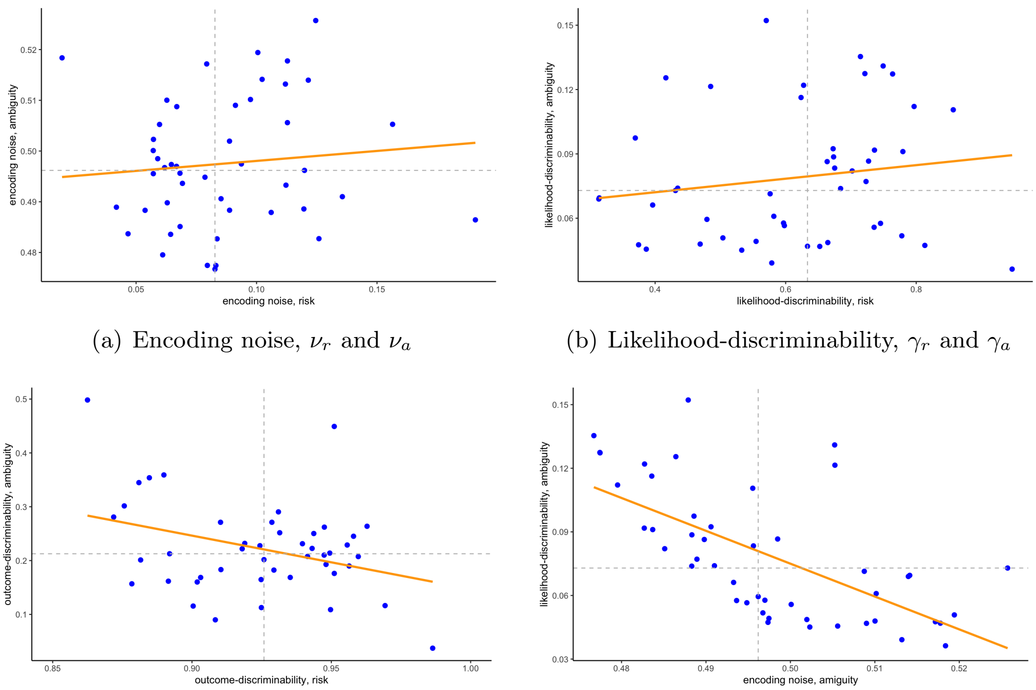
To test this prediction, I use an original dataset that contains a richer choice setup. The data contain observations for 47 subjects, and the data structure and experimental procedures closely follow those in L'Haridon et al. (2018). The stimuli are, however, richer in that the stimuli for risk are replicated exactly for ambiguity, including variation over outcomes and non-zero lower outcomes for both risk and ambiguity, thus allowing for the identification of utility curvature and outcome-discriminability under ambiguity as well as risk. Further details can be found in the online appendix.



(a) Densities of $\widehat{\gamma}_r^+$ and $\widehat{\gamma}_a^+$ (b) Densities of $\widehat{\alpha}_r^+$ and $\widehat{\alpha}_a^+$

Figure 5: Densities of $\widehat{\gamma}$ and $\widehat{\alpha}$ for risk and ambiguity

I start by testing the performance of the different models against each other. The NCM easily outperforms the PT specification (elpd difference: -3521.4 , SE: 21.3). Figure 5 shows density plots for $\widehat{\gamma}$ and $\widehat{\alpha}$, for risk and ambiguity. The difference between risk and ambiguity for $\widehat{\gamma}$, shown in panel 5(a), is sizeable. This sort of lowering of likelihood-sensitivity under ambiguity relative to risk, termed ambiguity-insensitivity, is well-documented in the literature (Abdellaoui et al., 2011; Dimmock et al., 2015; Trautmann and van de Kuilen, 2015; L'Haridon et al., 2018). Enke and Graeber (2019) have documented empirically that this phenomenon goes hand-in-hand with a lowering of the confidence people declare to have in their choices, which is consistent with the account presented here. The difference between outcome sensitivity for risk and ambiguity, displayed in panel 5(b), is much less pronounced. Nevertheless, the distribution under ambiguity appears to be slightly shifted to the left. This is confirmed by a Mann-Whitney test, indicating that outcome-sensitivity is indeed reduced under ambiguity ($p = 0.012$), thus resulting in a violation of PT.



(a) Encoding noise, ν_r and ν_a

(b) Likelihood-discriminability, γ_r and γ_a

(c) Outcome-discriminability, α_r and α_a

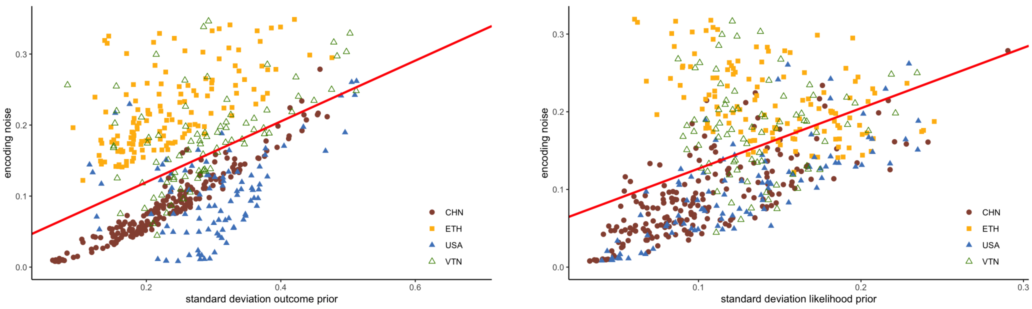
(d) Noise ν_a and likelihood-discriminability γ_a

Figure 6: NCM parameters under risk and ambiguity

Let us now take a look at the patterns emerging from the NCM, displayed in figure 6. Panel 6(a) shows a scatter plot of encoding noise for risk and ambiguity. The two are not significantly correlated. In accordance with the predictions discussed above, however, encoding noise for ambiguity is much larger than encoding noise for risk. This is in turn reflected in the other model parameters. In particular, likelihood-discriminability γ is much lower under ambiguity than under risk, as is apparent from panel 6(b). Similar results obtain for outcome-discriminability α, shown in panel 6(c). Once again discriminability is much lower under ambiguity than it is under risk. Notice also that this difference is much more pronounced than it was under PT, which can be traced back to the different definition of outcome distortions and its interaction with encoding noise. Finally, panel 6(d) shows the correlation between encoding noise and likelihood discriminability under ambiguity. The two are strongly negatively correlated ($\rho = -0.711, p < 0.001$), as is predicted by the NCM.

3.6 Is coding efficient?

My analysis so far was based on the noisy coding model I have derived. An interesting additional question, however, concerns whether behaviour may also conform to *efficient coding* predictions, according to which the noisiness in the likelihood function would optimally adapt to the prior given constraints on cognitive resources. To this end, I use the prediction formally derived by Frydman and Jin (2021), who show that under an efficient coding setup the encoding noise ought to covary with the standard deviation of the prior. While any such correlation in the data I use would not be causal—there is no random variation in the choice stimuli such as the one used by Frydman and Jin (2021)—such a correlation would nevertheless be indicative of the coding noise being efficient, since different *perceptions* of stimulus dispersion in the environment also ought to result in different encoding noise.



(a) Scatter plot of σ_o^+ and ν^+ (b) Scatter plot of σ_e^+ and ν^+

Figure 7: Scatter plot of prior SDs against encoding noiseL'Haridon and Vieider (2019)

Figure 7 shows correlations between the standard deviations of the outcome prior (panel 7(a)) and of the likelihood prior (panel 7(b)) with encoding noise. For parsimony, I only show the patterns for the usual four countries in the L'Haridon and Vieider (2019) data, and only for gains. Patterns for losses, the global data, or the data of Bruhin et al. (2010) are qualitatively similar. We observe a clear positive correlation between the standard deviation of the outcome prior and encoding noise ($\rho = 0.410, p < 0.001$), as well as between the standard deviation of the likelihood prior and encoding noise ($\rho = 0.513, p < 0.001$). This provides suggestive evidence that the coding noise on which the observed choice patterns are based is indeed *efficient*.

4 Discussion

Traditional models of decision-making under risk and uncertainty represent behaviour by means of deterministic models applied to objectively perceived stimuli. These deterministic models are subsequently augmented by an independently chosen stochastic model to allow for choice parameters to be recovered from noisy data. The noisy coding model presented in this paper turns this process on its head. Starting from the insight that the choice stimuli themselves may be encoded by noisy mental signals, I have shown how the optimal decoding of these signals by means of a mental prior may result in decisions that deviate systematically from the underlying optimal choice rule, and in particular, which may result in systematic outcome- and likelihood-distortions akin to those documented in the prospect theory literature. While predicting PT-like choice patterns, however, the model simultaneously predicts PT violations, in particular pertaining to the strict separation between the probabilistic and outcome dimensions under PT. It also suggests a novel interpretation of PT parameters, which may in turn yield better predictive performance, delivered for instance through the intuition that the mental prior may differ across different choice situations, which allows to better account for insurance and lottery play decisions than PT does based on standard parameter estimates (Sydnor, 2010).

Deviations from the normative benchmark of expected utility maximization such as predicted by the NCM are typically depicted as biases or rationality failures in economics. It is thus interesting to see how such patterns may result from a cognitive process which may be considered optimal. Indeed, the NCM builds on an optimal choice rule, which is based on accumulating information by means of Bayesian updating, and which entails expected value maximization. The deviations from the expected value benchmark purely arise from limited cognitive resources dedicated to a given decision problem, which will result in noise in the encoding of external stimuli, and hence regression to the mean of the mental prior. This suggests a resource-saving rationale which may be highly adaptive, and which is reminiscent of concepts of bounded rationality and resource-saving through adaptation as originally discussed by Herbert Simon (1955; 1959).

Relative to prospect theory, the NCM appears to hold some promise especially when it comes to the analysis of empirical data. For one, I have documented the predictive ability of the model to be vastly superior to the one of PT. The model also suggests that choice parameters may be malleable and amenable to manipulation, which could be exploited in policy making. When it comes to analyzing the correlates of behaviour under risk and uncertainty, the NCM furthermore provides a clear division of parameters pertaining to noisy cognition, as captured by the noisy mental encoding parameter, and those pertaining to the mental prior, which may incorporate more stable elements. This separation may help overcome issues that have hampered empirical analysis using PT, where several parameters usually co-vary in regression analysis, making an interpretation of their effects fraught with difficulties (see online appendix of L'Haridon and Vieider, 2019, for a number of examples).

An important question remains as to what may determine the prior itself. The neuroscience literature conceptually distinguishes two parts of the mental prior. One part can quickly adapt to a given choice situation, and is thus subject to experimental manipulation and study. Effects of priming, such as documented e.g. by Cohn, Engelmann, Fehr and Maréchal (2015), may be a manifestation of such an effect. The other part consists in a more stable component, which is much less studied in neuroscience. Conceivably, this component may be influenced by long-term experiences (Malmendier and Nagel, 2011), shaped by conscious education efforts (Doepke and Zilibotti, 2014; 2017), forged by environmental adaptation (Di Falco and Vieider, 2021), or be determined by intergenational transmission (Galor and Michalopoulos, 2012; Bouchouicha and Vieider, 2019). Investigating the process under which the more stable components of the mental prior is formed ought to be a priority area for future research.

The model proposed in this paper is more loosely linked to other models of adaptive preferences. Most notable is the connection to the evolutionary models of Robson (2001) and Netzer (2009), who use a fitness-maximizing model predicated on limited discernibility of outcomes to derive a utility function that adapts to the local environment. A common element is that all these models incorporate the intuition of just noticeable differences in utility, although in the NCM this is a result of the compressed mental representation of stimuli, whereas it is a modelling assumption in the Robson-Netzer framework, where it results in utility taking the form of a step function. Netzer et al. (2021) present a model in which an agent receives noisy signals about different lottery arms. The agent may thereby decide to oversample some lottery arms, which could lead to the overweighting of small probability events. While some of the underlying intuitions are similar to those developed in this model, the focus is different, with Netzer et al. (2021) primarily focusing on the question of what happens when decision noise goes to zero, which is complementary to the noisy coding approach presented in this paper.

The general framework based on the noisy neural coding of stimuli—and its extension to efficient coding schemes that allow for the endogenization of the likelihood function—presents the promise of a unifying theory of individual choice behaviour. On the one hand, encoding noise plays a central role in the model, driving deviations from optimal behaviour, such as expected value maximization for small-stake risks. This results in behavioural regularities that are likely to impact decisions far beyond the particular setup used in this paper. For instance, we may well expect the presentation format of stimuli to impact choices, and the noisiness of stimulus encoding may be expected to increase systematically with the difficulty of the choice tasks. The precise implications of this insight deserve close attention, and are thus left for future work.

5 Conclusion

I proposed a model of decision making under uncertainty resting on the noisy coding of external stimuli. These stimuli are subsequently optimally decoded using a mental prior to result in actionable decision inputs. I showed that this setup results in a linear in log-odds specification of a popular probability-distortion function, whereby subjectively distorted likelihood ratios are traded off against a subjectively distorted cost-benefit ratio. The noisy neural coding model derived in this paper can thus be seen as a generative model of prospect theory preferences. At the same time, the model makes predictions that suggest that prospect theory will be violated in certain situations due to its strong separability precepts between outcome and probability representations. Taking the model to a variety of data sets, I have shown that the noisy coding model outperforms prospect theory significantly in terms of predictive performance. This superior performance of the noisy coding model can be traced back to its stochastic nature. While under prospect theory decision noise is added onto the deterministic model and is generally assumed to be orthogonal to the latter, in the noisy coding model decision noise and the core model parameters result from one and the same process, thus being inextricably linked. This interaction between noise and decision parameters appears to be a central feature of data on decisions under uncertainty, explaining the superior performance of the noisy coding model.

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SUPPLEMENTARY MATERIALS (For online publication)

Noisy Neural Coding and Decisions under Uncertainty

S1 Derivation of the noisy coding model

S1.1 Derivation details

In this section, I provide additional details about the derivations underlying the equations shown in the main text. Combining the likelihoods in equation 4 in the main text with the priors in equation 5 in the main text by Bayesian updating, we obtain the following posterior distributions conditional on the mental signals:

$$ln\left(\frac{P[e]}{P[\tilde{e}]}\right)\Big|r_e \sim \mathcal{N}\left(\frac{\sigma_e^2}{\sigma_e^2+\nu_e^2}\times r_e + \frac{\nu_e^2}{\sigma_e^2+\nu_e^2}\times ln\left(\frac{\widehat{P}[e]}{\widehat{P}[\tilde{e}]}\right), \frac{\nu_e^2\sigma_e^2}{\nu_e^2+\sigma_e^2}\right) \tag{16}$$

$$ln\left(\frac{c-y}{x-c}\right)\Big|r_o \sim \mathcal{N}\left(\frac{\sigma_o^2}{\sigma_o^2+\nu_o^2}\times r_o + \frac{\nu_o^2}{\sigma_o^2+\nu_o^2}\times ln\left(\frac{\widehat{k}}{\widehat{b}}\right), \frac{\nu_o^2\sigma_o^2}{\nu_o^2+\sigma_o^2}\right). \tag{17}$$

where we define $\gamma \triangleq \frac{\sigma_e^2}{\sigma_e^2+\nu_e^2}$ and $\alpha \triangleq \frac{\sigma_o^2}{\sigma_o^2+\nu_o^2}$. It follows that $\frac{\nu_e^2}{\sigma_e^2+\nu_e^2} = 1-\gamma$ and $\frac{\nu_o^2}{\sigma_o^2+\nu_o^2} = 1-\alpha$. Taking the expressions multiplying the logarithms to the exponent and further defining $\xi \triangleq \left(\frac{\widehat{P}[e]}{\widehat{P}[\tilde{e}]}\right)^{1-\gamma}$ and $\zeta \triangleq \left(\frac{\widehat{k}}{\widehat{b}}\right)^{1-\alpha}$ gives the posterior expectations shown in equation 6 in the main text.

We can then substitute the posterior expectations into the mental choice rule in equation 7 to obtain the threshold equation 8 in the main text. Given that r_e and r_o follow a normal distribution, their weighted difference in equation 8 will itself follow a normal distribution with an expectation equal to the weighted difference of the means of the distributions of r_e and r_o:

$$\gamma\times r_e - \alpha\times r_o \sim \mathcal{N}\left(\gamma\times ln\left(\frac{P[e]}{P[\tilde{e}]}\right) - \alpha\times ln\left(\frac{c-y}{x-c}\right), \omega^2\right), \tag{18}$$

where $\omega \triangleq \sqrt{\gamma^2\nu_e^2+\alpha^2\nu_o^2}$ represents the standard deviation of the weighted difference of noisy mental signals. This yields the following z-score:

$$z = \frac{\gamma\times r_e - \alpha\times r_o - \left[\gamma\times ln\left(\frac{P[e]}{P[\tilde{e}]}\right) - \alpha\times ln\left(\frac{c-y}{x-c}\right)\right]}{\omega}, \tag{19}$$

which follows a standard normal distribution. Comparing this z-score to an equivalent z-score for equation 8 yields the probabilistic choice rule in equation 9.

S1.2 Derivation for unlogged choice rule

The derivations in the main text as well as the details in the last subsection were based on the logarithmic choice rule in equation 3. If we were to use the median of the posterior instead of its mean in the choice rule, then the derivations would be identical if we were to base them on the unlogged choice rule in equation 1 instead. If we want to use the posterior expectation in the choice rule instead of the median, as done here, then we need to obtain the posterior means of the unlogged quantities by taking the exponential of the expectations in equation 6 in the main text:

$$E\left[\left(\frac{P[e]}{P[\tilde{e}]}\right)|r_e\right] = e^{\gamma\times r_e + ln(\xi) + \frac{1}{2}\sigma_e^2} \ , \ E\left[\left(\frac{c-y}{x-c}\right)|r_o\right] = e^{\alpha\times r_o + ln(\zeta) + \frac{1}{2}\sigma_o^2}, \tag{20}$$

which obtain by the properties of the mean of the log-normal distribution. We can now substitute these quantities into the following mental choice rule, which is based on the unlogged optimal decision rule in equation 2:

$$E\left[\frac{P[e|s]}{P[\tilde{e}|s]}|r_e\right] > E\left[\frac{(c-y)}{(x-c)}|r_o\right], \tag{21}$$

to obtain:

$$e^{\gamma\times r_e + ln(\xi) + \frac{1}{2}\sigma_e^2} > e^{\alpha\times r_o + ln(\zeta) + \frac{1}{2}\sigma_o^2}. \tag{22}$$

Taking the logarithm of both sides, defining $\widehat{\xi} \triangleq e^{ln(\xi) + \frac{1}{2}\sigma_e^2}$ and $\widehat{\zeta} \triangleq e^{ln(\zeta) + \frac{1}{2}\sigma_o^2}$, we obtain:

$$\gamma \times r_e - \alpha \times r_o > ln(\widehat{\delta})^{-1}, \tag{23}$$

where $\widehat{\delta} \triangleq \widehat{\xi} \times \widehat{\zeta}^{-1}$. All further derivations proceed like above. The sole difference with the results presented in the main text thus flow from the difference in definition of $\widehat{\delta}$ versus δ. That is, the variance of the priors will enter into the definition of $\widehat{\delta}$, while it will not enter the definition of δ. In the main text, I argued that the logged choice rule appears more plausible than its unlogged version for computational reasons. The less intuitive formulation of the prior emerging from the unlogged choice rule may constitute a further argument for my preferred setup. That being said, the difference between the two setups is *quantitative* in nature, rather than *qualitative*, and it does not affect the main conclusions drawn in the paper.

S2 Additional results

This section presents additional details and results for the analysis of 2-outcome wagers under risk, both under PT and the NCM. The structure follows roughly the one followed in the main text. Notice that the basic econometric setup in Bruhin et al. (2010) and L'Haridon and Vieider (2019) are the same, as are the type of stimuli used in the two papers, which will allow me to discuss the econometric approach in one unified setup. Whereas Bruhin et al. (2010) applied such a setup to a finite mixture model, and L'Haridon and Vieider (2019) estimated a (frequentist version of a) random preference model, I will use the basic setup to estimate a Bayesian random preference (or hierarchical) model.

S2.1 Estimation of the Bayesian hierarchical model

I estimate individual-level parameters using Bayesian hierarchical models (Gelman and Hill, 2006; Gelman et al., 2014b; McElreath, 2016). Take a parameter vector $\boldsymbol{\theta}_i$, indicating individual-level parameters. This vector follows the following distribution:

$$\boldsymbol{\theta}_i \sim \mathcal{N}(\overline{\boldsymbol{\theta}}, \Sigma), \tag{24}$$

where $\overline{\boldsymbol{\theta}}$ is a vector of hyperparameters containing the means of the individual-level parameters, and Σ is a variance-covariance matrix of the individual-level parameters. I estimate the parameters in Stan launched from Rstan (Stan Development Team, 2017). Estimations typically employ 4 chains with 2000 iterations per chain, of which 1000 are warmup iterations—the default settings of Stan (in some cases iterations may be increased because of efficiency concerns in the estimated parameters). I check convergence by examining the R-hat statistics, and by checking for divergent iterations. The estimation algorithm was developed using simulations, and making sure that I could recover individual-level parameters from those simulations. The model endogenously estimates the priors for the individual-level parameters from the aggregate data, resulting in partial pooling. This is indeed a central strength of the model, which tends to reduce issue with overfitting of few observations. The priors for the estimation of the aggregate-level means are chosen in such a way as to be mildly regularizing (McElreath, 2016). That is, their variance is chosen in such a way that all plausible parameters fall into a region attributed high likelihood, but narrow enough to nudge the simulation algorithm towards convergence. In any case, the datasets I use have sufficient data at the aggregate level for the priors chosen to have little or no impact on the final result.

Following the estimation approach used in both paper and the structure of the data, I use directly the density around the observed switching point to estimate the model. While this deviates somewhat from the discrete choice model presented in the main text for the NCM, using such a model instead does not affect the results in any way, since both datasets push participants towards single switching. This is indeed the sense in which certainty equivalents of the type used in the data are not specifically geared towards estimating the NCM, being rather driven by the deterministic conceptions underlying PT. In this sense, the test of the NCM against PT is conservative in nature, since the data tend to naturally favour PT. For the PT model, the model thus takes the following form:

$$ce \sim \mathcal{N}(u^{-1}[w(p)u(x) + (1 - w(p))u(y)], \widehat{\omega}^2 \times |x - y|), \tag{25}$$

with $u(x) = x^{\widehat{\alpha}}$ and $w(p) = \frac{\widehat{\delta} p^{\widehat{\gamma}}}{\widehat{\delta} p^{\widehat{\gamma}} + (1-p)^{\widehat{\gamma}}}$, and where multiplying the variance $\widehat{\omega}$ by $|x - y|$ allows for heteroscedasticity across choice lists with different step sizes between choices, following the approach in the original papers from which I took the data. Contextualizing choices by letting the error be heteroscedastic across to the *utility difference*, $|u(x) - u(y)|$, such as proposed by Wilcox (2011), does not affect the conclusions I draw in any way.

The priors are chosen such as to be informative about the expected location of the model parameters, without imposing any undue restrictions on the data. This is typically referred to as *mildly regularizing* priors, and it helps convergence in the model. For instance, the prior chosen for $\widehat{\gamma}$ has a mean of 0.7 on the original scale, with 95% of the probability mass allocated to a range of $[0, 3.92]$. This can be expected to encompass most likely parameter values. Given furthermore the large quantity of data present at the aggregate level, the data can easily overpower the prior even for parameters falling outside this range. Making the prior more diffuse and shifting the mean to e.g. 1, does not affected the estimated parameters in any way, showing that the prior has only a minimal influence on the ultimate parameter estimates.

For the NCM I proceed similarly, but I transform the data to obtain a 'dual-EU decision weight', $dw \triangleq \frac{ce-y}{x-y}$, which is distributed as follows:

$$dw \sim \mathcal{N}\left(\frac{\delta^{\frac{1}{\alpha}} p^{\frac{\gamma}{\alpha}}}{\delta^{\frac{1}{\alpha}} p^{\frac{\gamma}{\alpha}} + (1-p)^{\frac{\gamma}{\alpha}}}, \nu \times \sqrt{\alpha^2 + \gamma^2}\right), \tag{26}$$

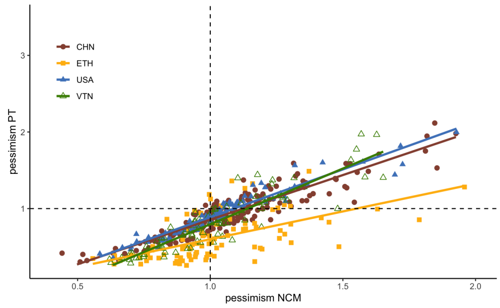
where the parameters are defined as in the model in the text, i.e. they are derived from the underlying parameters $\nu, \sigma_p, \sigma_o, \psi$. Given that much less is known about these parameters, I have generally chosen priors with a mean of 0.5 and a larger variance, in order not to preclude any but the most extreme estimates ex ante. As above, the data are sufficient to overcome this prior if necessary, and changing the specifications of the prior to make it more diffuse does not affect the estimates.

S2.2 The NMC generates PT functionals: Additional results

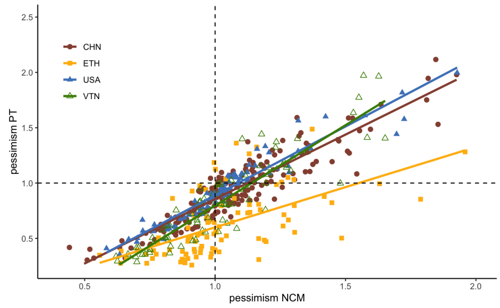
In what follows, I present additonal results illustrating the generative nature of NCM for PT functionals. Figure S8 shows the correlations between the NCM-derived PT parameters and the true PT parameters for losses in the L'Haridon and Vieider (2019) data. The results for losses track those for gains closely.

I start again by discussing the correlation between decision noise in the NCM with the SD of the additive noise term in PT, shown in panel 8(a). The association between the two variables is extremely tight ($\rho = 0.909, p < 0.001$), as one might expect given the fact that both are identified from the noisiness of choices. Given that ω^- contains in itself the model parameters α^- and γ^-, whereas $\widehat{\omega}$ is implemented as an independent dimension, we would however expect larger differences to emerge on those parameters. These differences start surfacing in panel 8(b), showing the correlation of the optimism parameter emerging from the two models. At $\rho = 0.862, p < 0.001$, they are again highly correlated. Indeed, δ^- is only indirectly influenced by the decision noise encapsulated in ω^-. Nevertheless, this indirect influence surfaces in the narrower distribution the parameter has under the NCM compared to the PT model (IQR of 0.522 for $\widehat{\delta}^-$ versus 0.276 for δ^-), and is a manifestation of the shrinking power of $1-\gamma^-$ in the NCM.

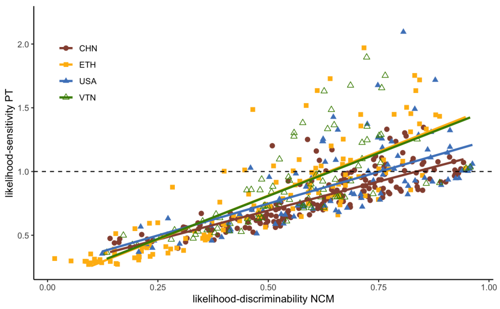
The parameter correlations barely decrease once we move to likelihood distortions, shown in panel 8(c), and remain highly significant ($\rho = 0.827, p < 0.001$). As already witnessed for gains, values of $\widehat{\gamma}^- > 1$ estimated using the PT specification do not necessarily map into values of γ^- close to 1. This reflects a central mechanism of the NCM



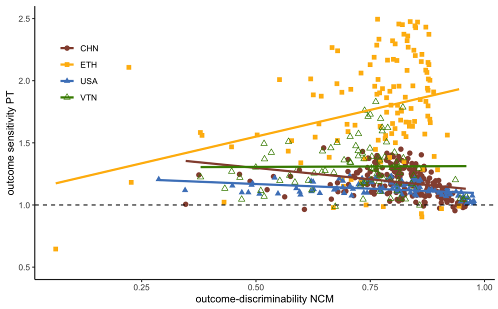
(a) Density outcome disciminability, α



(b) Density probabiliity disciminability, γ



(c) Density optimism



(d) Density decison noise

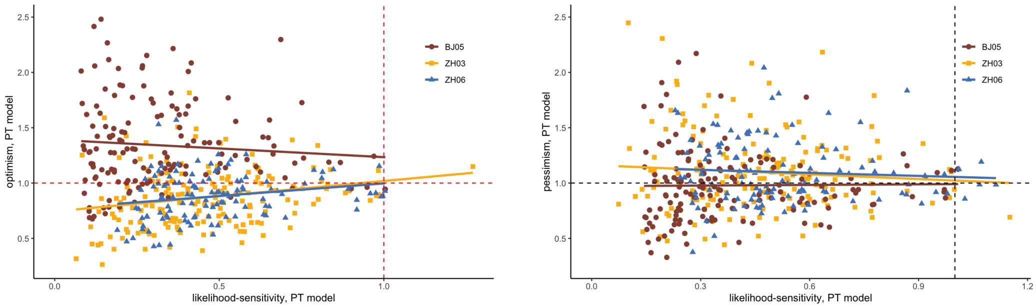
Figure S8: NCM-generated parameters and PT parameters for losses in L'Haridon and Vieider (2019)

at work. Values of likelihood-sensitivity larger than 1 under PT are typically associated with high levels of noise. Given the close association between decision noise and γ in the NCM, those choice patterns are best reflected by increased decision noise, which in turn results in values of $\gamma < 1$. It is this which drives the superior predictive performance of the NCM. Outcome distortions are shown in panel 8(d), where as for gains we find no correlation between the PT and NCM parameter.

S2.3 PT parameter correlations: Additional results

This section adds some further results to the correlations amongst PT parameters presented in the main text. I will present additional results for both Bruhin et al. (2010) and L'Haridon and Vieider (2019), and for losses as well as for gains.

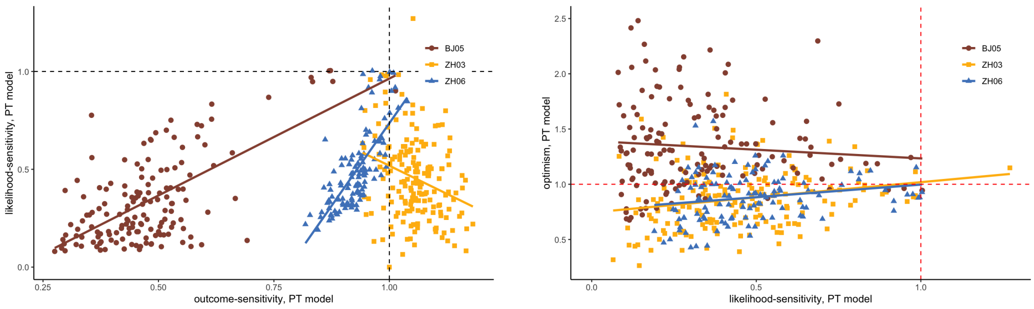
I start by presenting correlations between additional PT parameters in the Bruhin et al. (2010) data. Another interesting pattern emerges from the relation between likelihood-sensitivity and optimism for gains, shown in figure S9 panel 9(a), and between likelihood-sensitivity and pessimism for losses, shown in panel 9(b). In both cases, the values of $\widehat{\delta}$ are most dispersed for small values of $\widehat{\gamma}$, with the dispersion decreasing



(a) Scatter plot of $\widehat{\gamma}^{+}$ and $\widehat{\delta}^{+}$ (b) Scatter plot of $\widehat{\gamma}^{-}$ and $\widehat{\delta}^{-}$

Figure S9: Scatter plot of PT parameters $\widehat{\gamma}$ and $\widehat{\delta}$
The parameters have been obtained from the estimation of a PT model plus additive noise. The different colours and shapes represent the 3 experiments in Bruhin et al. (2010): ZH03 stands for Zurich 03; ZH6 for Zurich 06; and BJ05 for Beijing 05.

markedly as $\widehat{\gamma}$ increases, resulting in a funnel with the narrow part pointing to the right. Testing the correlation of absolute deviations of $\widehat{\delta}$ from 1 with $\widehat{\gamma}$, I find highly significant effects for both gains ($\rho = -0.284, p < 0.001$) and losses ($\rho = -0.264, p < 0.001$). These patterns have no obvious explanation under PT. In the NCM, however, they are predicted by the definition of $\delta = \left(\frac{\psi}{1-\psi}\right)^{1-\gamma}$. That is, larger values of γ shift the attention from the prior to the likelihood, thus compressing δ towards 1. Panel 10(a) visualizes the correlations between $\widehat{\alpha}$ and $\widehat{\gamma}$ already discussed in the main text.



(a) Scatter plot of $\widehat{\alpha}$ against $\widehat{\gamma}$ (b) Scatter plot of $\widehat{\gamma}$ against $\widehat{\delta}$

Figure S10: Correlations between PT parameters in Bruhin et al. (2010)

Panel 10(b) further shows correlational patterns between $\widehat{\gamma}$ and $\widehat{\delta}$. According to the NCM, the results should depend on the initial value of ψ. In particular, the larger the value of $\widehat{\gamma}$, the closer the value of $\widehat{\delta}$ should be compressed towards 1. This is exactly what we observe. In experiments where we observe preponderantly values of $\widehat{\delta} < 1$, the distance to 1 decreases as $\widehat{\gamma}$ increases, thus resulting in a positive correlation between the two parameters. This is the case in the Zurich 03 experiment ($\rho = 0.262, p <$

0.001), as well as in the Zurich 06 experiment ($\rho = 0.369, p < 0.001$). In the Beijing 05 experiment, on the other hand, we observe very large values of $\widehat{\delta} > 1$. We may thus expect a negative relationship with $\widehat{\gamma}$. We fail to observe such a relationship in the data ($\rho = 0.045, p = 0.54$). This may be due to the fact that we have very few observations with large likelihood-sensitivity in that experiment.

I next document correlations amongst the PT parameters in the global data of L'Haridon and Vieider (2019), for all 30 countries and 3000 subjects. Different countries are not distinguished by colour or symbols as they were before, since there are not enough colours and symbols to render such a distinction intelligible. Correlations are tested on parameters demeaned at the country level, corresponding to a fixed effects specification.

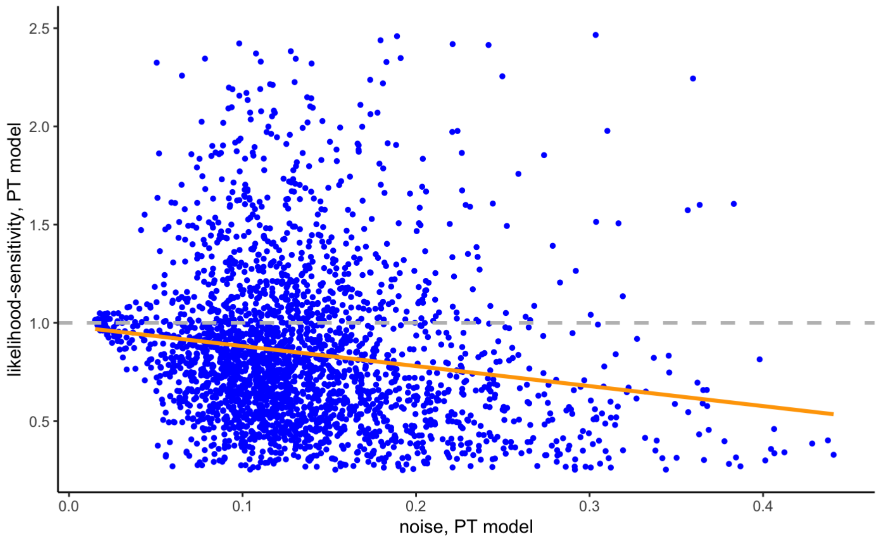


Figure S11: Correlatiion of $\widehat{\omega}^+$ and $\widehat{\gamma}^+$ in L'Haridon and Vieider (2019)

Figure S11 shows the correlation between noise and likelihood-sensitivity for gains under PT. We find the usual negative correlation already described for the risk and rationality data ($\rho = -0.2, p < 0.001$). At first, the negative correlation may appear somewhat weaker than witnessed for Bruhin et al. (2010). Closer examination reveals that this is due to the presence of a larger number of estimates of $\widehat{\gamma}^+ > 1$. As already described for utility curvature in the main text, both negative and positive deviations tend to be correlated with noise for parameters estimated in a PT context. Taking absolute deviations of likelihood sensitivity from 1, $|1 - \widehat{\gamma}^+|$, the correlation becomes indeed much stronger ($\rho = 0.386, p < 0.001$).

The patterns for losses are very similar, and are shown in figure S12. Once again,

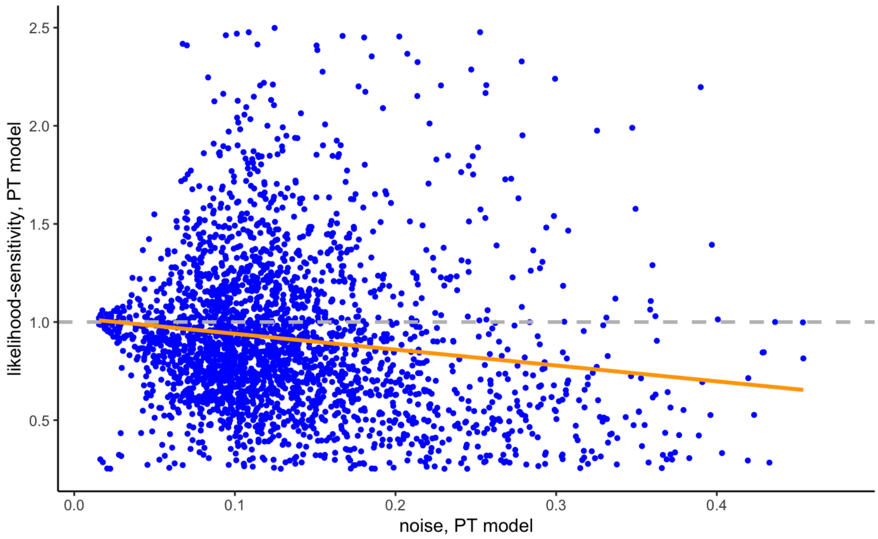


Figure S12: Correlatiion of $\widehat{\omega}^-$ and $\widehat{\gamma}^-$ in L'Haridon and Vieider (2019)

we observe a negative correlation in the global data ($\rho = -0.167, p < 0.001$), albeit one that is not as strong as we might have expected based on the Bruhin et al. (2010) results. This is once again driven by the presence of values of $\widehat{\gamma}^- > 1$, which are also associated with high noise levels. Looking at the correlations between noise and absolute deviations of the variable from 1, $|1 - \widehat{\gamma}^-|$, the correlation thus appears much stronger ($\rho = 0.445, p < 0.001$).

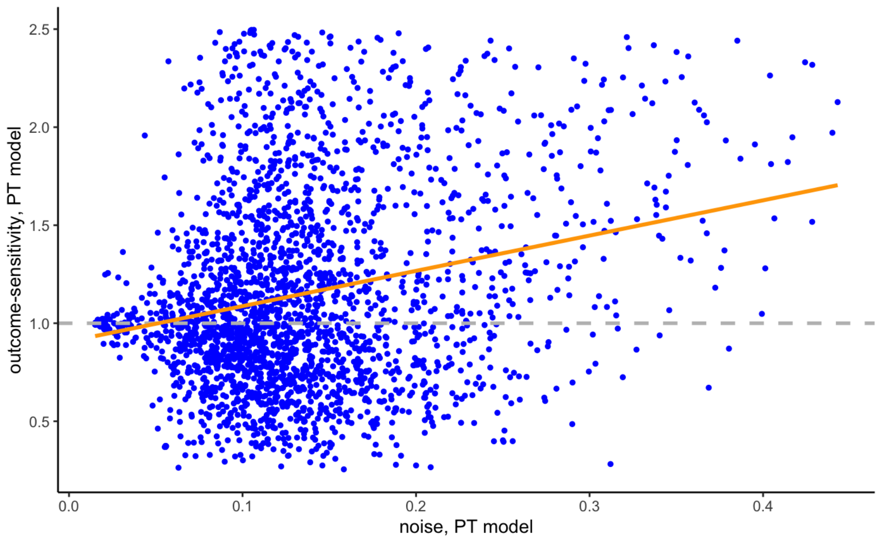


Figure S13: Correlatiion of $\widehat{\omega}^+$ and $\widehat{\alpha}^+$ in L'Haridon and Vieider (2019)

Figure S13 shows the global correlation between noise and outcome sensitivity for gains. The correlation in the global data is positive, reflecting the fact that the median outcome sensitivity parameter is positive at 1.125. The correlation is highly significant ($\rho = 0.141, p < 0.001$). Results for losses are shown in figure S14, where the same patterns appear to be even more accentuated ($\rho = 0.34, p < 0.001$).

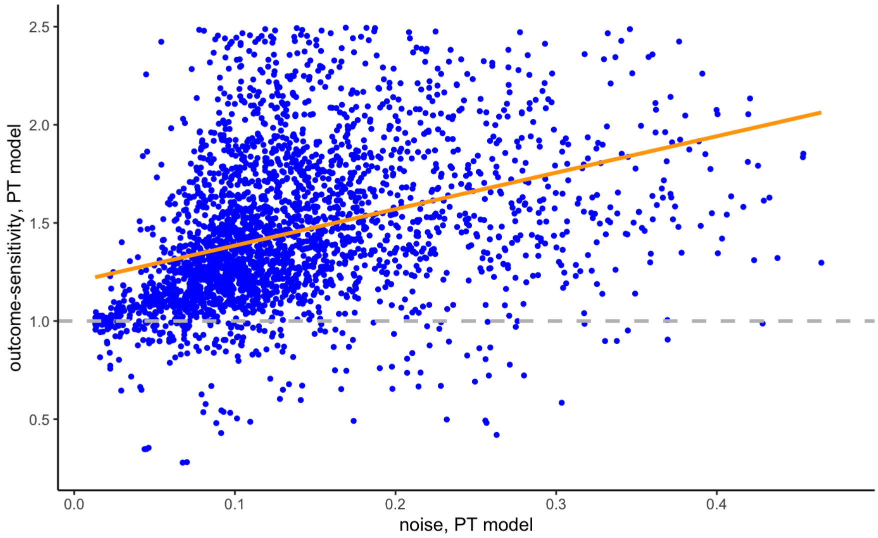


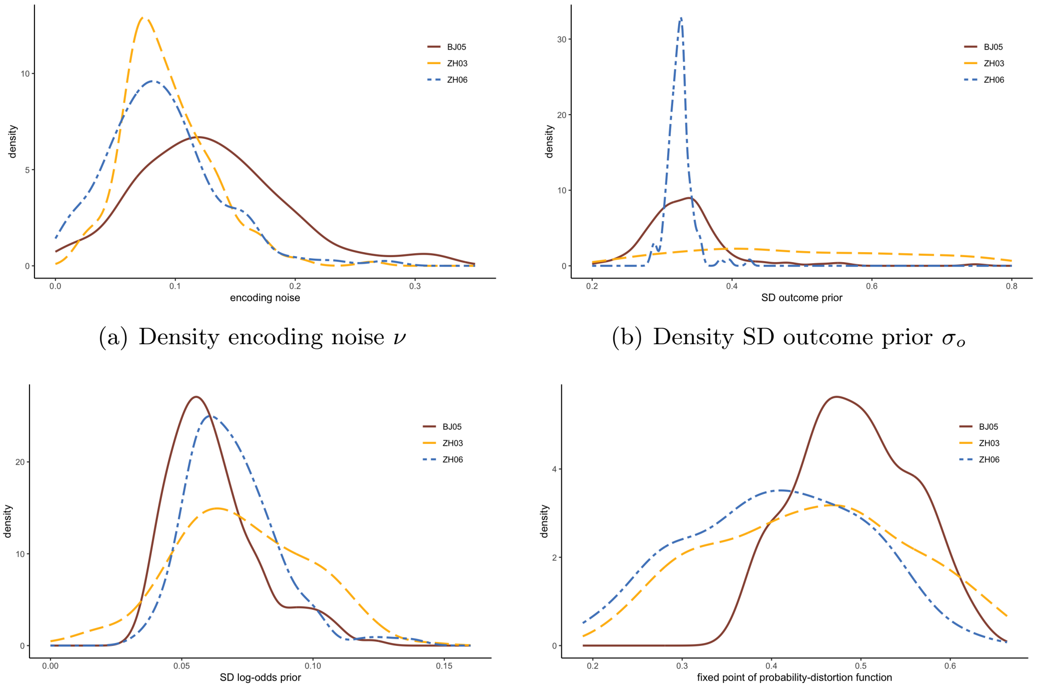
Figure S14: Correlatiion of $\widehat{\omega}^+$ and $\widehat{\alpha}^+$ in L'Haridon and Vieider (2019)

S2.4 Distribution of NCM and NCM-generated parameters

In this section, I show the distribution of the NCM parameters. This concerns both the native NCM parameters, and the NCM-derived PT parameters.

Let us now take a look at the native NCM parameters ν, ψ, σ_e and σ_o. Figure S15 shows the density functions for the 4 NCM parameters for gains for the Bruhin et al. (2010) data. The parameters are distributed over reasonable ranges, and resemble each other for the different experiments. The largest differences between experiments emerge for the standard deviation of the outcome prior, σ_o, in panel 15(b) and for the fixed point of the probability-distortion function, ψ, in panel 15(d). The first has a relatively broad distribution in the Zurich 03 experiment. The second tends towards higher values in the Beijing 05 experiment.

I next plot those same parameters for the data of L'Haridon and Vieider (2019), once again for gains and for the usual 4 countries. The parameter distributions can be seen in figure S16. Panel 16(a) shows systematic differences in encoding noise between countries, on top of the differences between individuals within each country. In particular, the US and China have relatively low encoding noise, while Ethiopia has very large



(a) Density encoding noise ν

(b) Density SD outcome prior σ_o

(c) Density SD probabillity prior σ_e

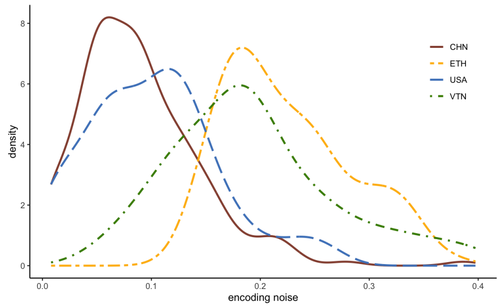
(d) Density fixed point probability-distortions ψ

Figure S15: Density functions of NCM parameters for Bruhin et al. (2010)

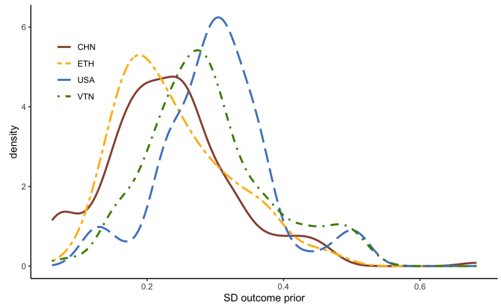
encoding noise, with Vietnam falling in between. Panel 16(d) also shows systematic differences between countries, with Vietnam and Ethiopia having larger fixed points of the probability-distortion function on average. The between-country differences for the two scale parameters of the prior, shown in panel 16(b) and panel 16(c) are somewhat less pronounced.

Figure S17 shows the density functions for the 4 NCM parameters for losses estimated from the data of Bruhin et al. (2010). The parameters resemble each other for the different experiments. The largest differences between experiments emerge once more for the standard deviation of the outcome prior, σ_o^-, and for the fixed point of the probability-distortion function, ψ^-. The first has a relatively broad distribution in the two experiments in Zurich. The second tends towards higher values in the Beijing 05 experiment, indicating higher fixed points of the probability-distortion function, just as observed for gains. Encoding noise is somewhat higher in Zurich 06 and Beijing 05 when compared to Zurich 03.

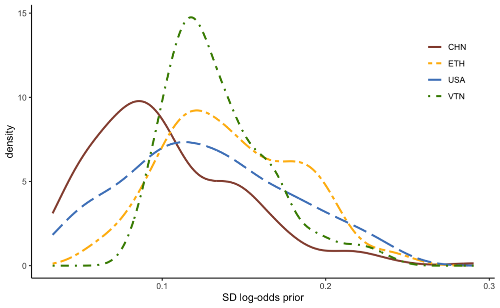
I plot the NCM parameter distributions for losses in the data of L'Haridon and Vieider (2019) in figure S18 for the usual 4 countries. Panel 18(a) shows systematic dif-



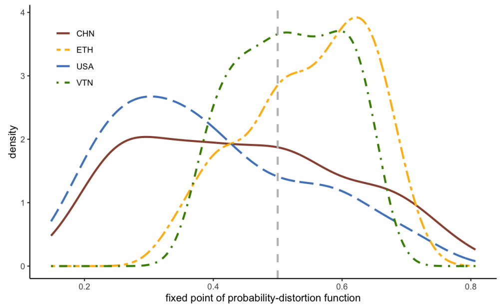
(a) Density encoding noise ν



(b) Density SD outcome prior σ_o

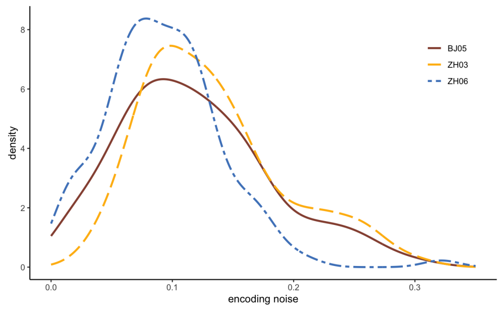


(c) Density SD probabillity prior σ_e

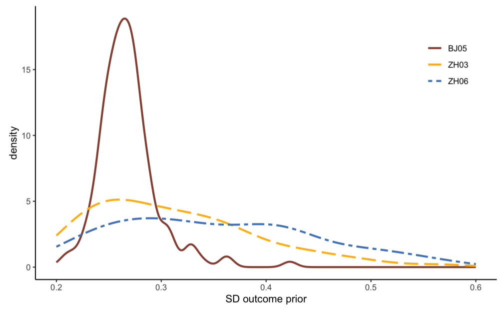


(d) Density fixed point probability-distortions ψ

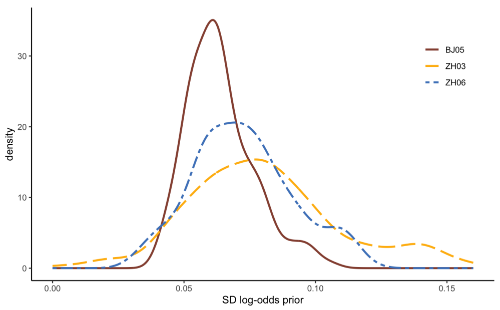
Figure S16: Density functions of NCM parameters for L'Haridon and Vieider (2019)



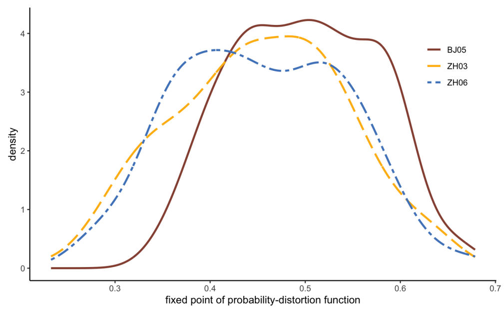
(a) Density encoding noise



(b) Density SD outcome prior



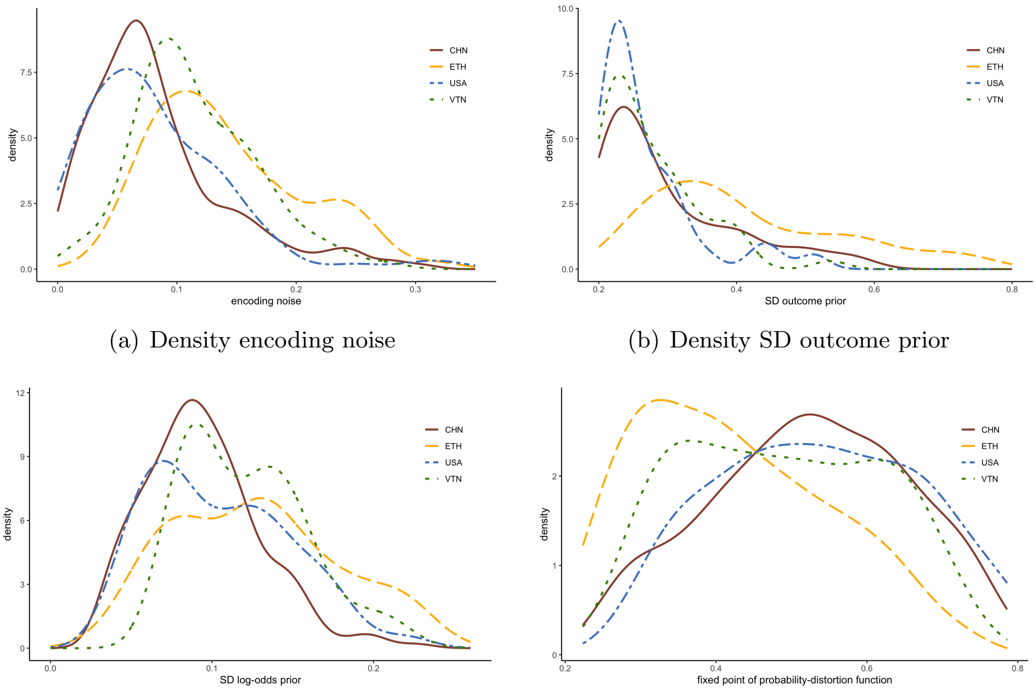
(c) Density SD probabillity prior



(d) Density fixed point probability-distortions

Figure S17: Density functions of NCM parameters for losses in Bruhin et al. (2010)

ferences in encoding noise between countries, on top the differences between individuals within each country. These differences are closely related to those observed for gains. In particular, the US and China have relatively low encoding noise, while Ethiopia and Vietnam have larger levels of encoding noise. Panel 18(d) also shows systematic differences between countries, with Vietnam and Ethiopia having somewhat fixed points of the probability-distortion function on average, thus inverting the pattern see for gains. The between-country differences for the two scale parameters of the prior are somewhat less pronounced, except for the somewhat diffuse distribution of the outcome prior variance in Ethiopia.



(a) Density encoding noise

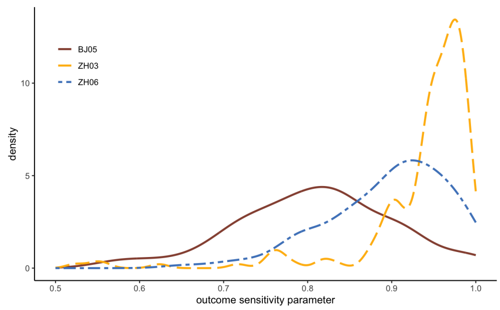
(b) Density SD outcome prior

(c) Density SD probabillity prior

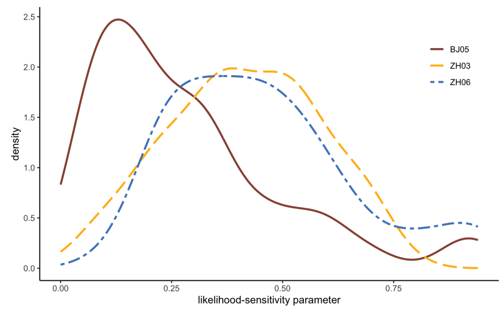
(d) Density fixed point probability-distortions

Figure S18: Density functions of NCM parameters for losses in L'Haridon and Vieider (2019)

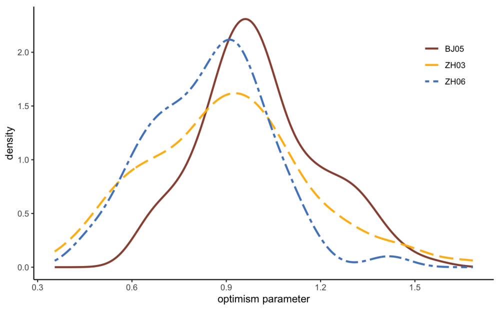
Let us now have a look at the distributions of the NCM-generated PT parameters, α, γ, δ, and ω. Figure S19 shows the density functions for gains in the data of Bruhin et al. (2010). The parameter values look reasonable, and fall into ranges we may expect to emerge from a PT model. Panel 19(a) and panel 19(b) show the largest differences between experiments, concerning outcome-discriminability α and likelihood-discriminability γ, respectively. Participants in Zurich in 2003 appear to be particularly discriminating towards outcomes, while Chinese participants in 2005 show particularly low discriminability in the likelihood dimension. Differences across experiments between other parameters tend to be less pronounced, even though optimism δ tends to be larger in the Beijing 05 experiment than in the two experiments in Zurich. Decision noise ω is virtually indistinguishable across the three experiments.



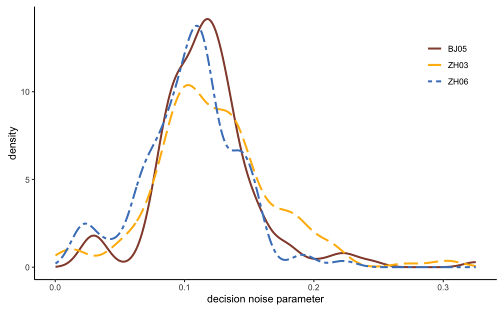
(a) Density outcome-discriminability α^+



(b) Density likelihood-discriminability γ^+



(c) Density optimism δ^+

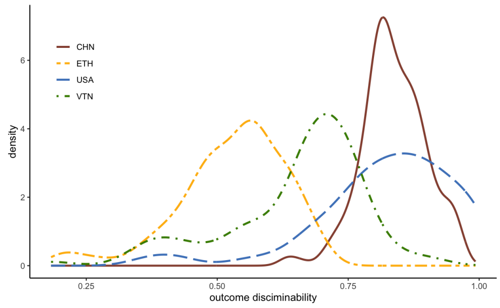


(d) Density decison noise ω^+

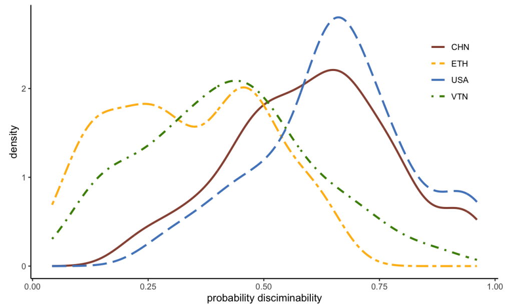
Figure S19: Density functions of NCM-generated PT parameters for Bruhin et al. (2010)

Figure S20 shows the same distributions for the four countries extracted from the L'Haridon and Vieider (2019) data, once again for gains. Panel 20(a) shows the distribution of outcome-discriminability separately for the four countries. Discriminability is highest in the US and China, significantly lower in Vietnam, and lower even in Ethiopia. Very similar patterns are observed for likelihood-discriminability, displayed in panel 20(b). Again discriminability is highest in the US and China, with Vietnam and Ethiopia displaying significantly lower levels. This is hardly surprising, given the important role of encoding noise ν in the makeup of both parameters. Indeed, encoding noise is highly correlated with both α and γ ($\rho = -0.872$ and $\rho = -0.842$, respectively, with $p < 0.001$ in both cases), as one would expect based on the modelling setup and parameter definitions.

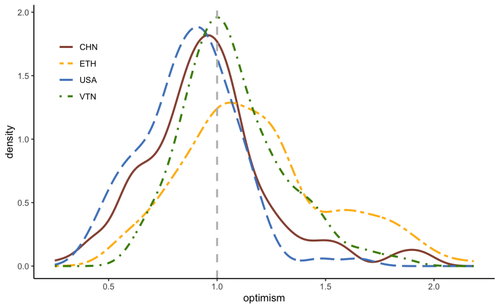
Similarly systematic patterns are shown in panel 20(c) for optimism, and in panel



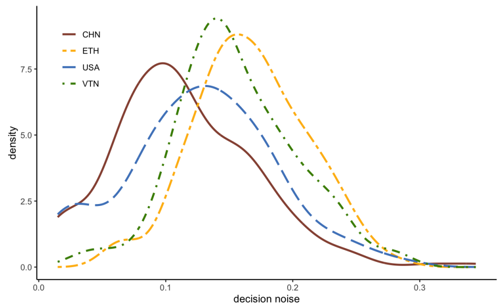
(a) Density outcome-disciminability, α^+



(b) Density likelihood-disciminability, γ^+



(c) Density optimism δ^+

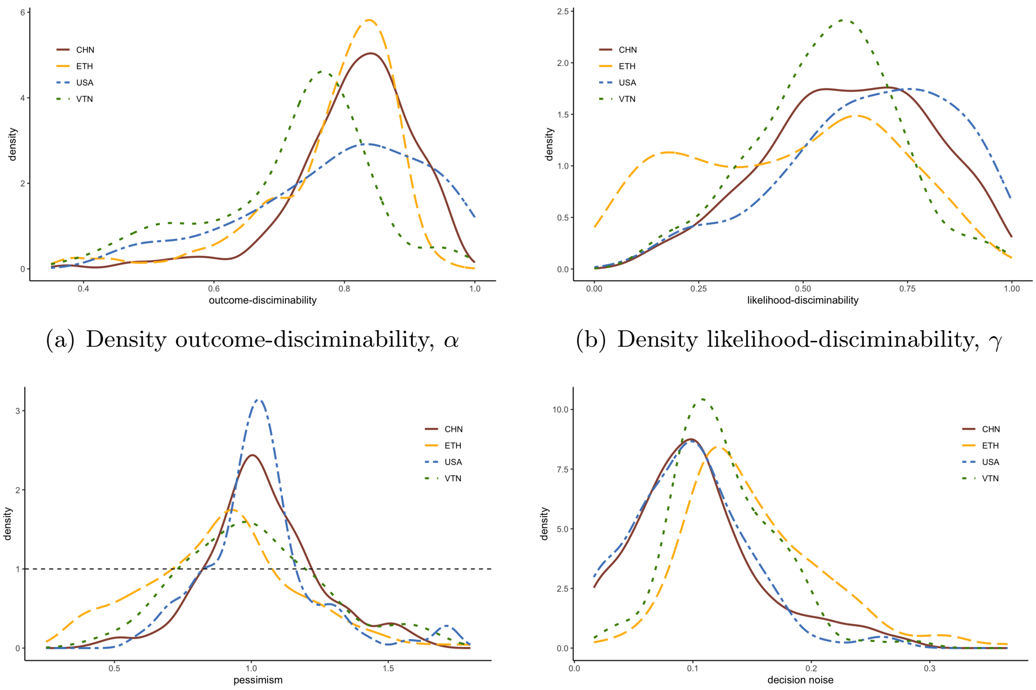


(d) Density decison noise ω^+

Figure S20: Density functions of NCM-generated parameters for L'Haridon and Vieider (2019)

20(d) for decision noise. Optimism is lowest in the US data, closely followed by China. Vietnam and especially Ethiopia show considerably higher levels of optimism. The patterns documented for for these parameters are also reflected in differences in decision noise ω^+. In particular, decision noise is is markedly higher in Vietnam than it is in China or the US, with the US data exhibiting higher noise than the Chinese data. Decision noise is, however, highest in the Ethiopian data. The similarities in the differences across the parameters are indeed unsurprising, given that the parameters α and γ, which in turn are determined by ν, feed back into decision noise ω.

Figure S21 shows the distributions for the four countries extracted from the L'Haridon and Vieider (2019) data for losses. Panel 21(a) shows the distribution of outcome-discriminability separately for the four countries. Discriminability is highest in China, with the US having a broader distribution, i.e. more individual heterogeneity, while being lowest in Vietnam. Very similar patterns are observed for likelihood-discriminability, displayed in panel 21(b). Discriminability is highest in the US and China, with Vietnam and especially Ethiopia displaying lower levels. The same systematic patterns are shown in panel 21(c) for pessimism, and in panel 21(d) for decision noise. Once again, the between-country differences in optimism are more muted than one might have expected from the differences in the fixed points of the function displayed in panel 18(d), which ones again happens due to the systematic differences in likelihood discriminability entering the definition of the latter. The same happens for decision noise ω^-. Although decision noise is higher in Vietnam and Ethiopia than it is in the US or China, the differences are less pronounced than those observed for encoding noise ν^-. This happens because the decision parameters α^- and γ^-, which are determined by ν^-, feed back into decision noise ω^-, compensating for the impact of ν^-.



(a) Density outcome-disciminability, α

(b) Density likelihood-disciminability, γ

(c) Density optimism

(d) Density decison noise

Figure S21: Density functions of NCM-generated parameters for losses in L'Haridon and Vieider (2019)

S2.5 Testing NCM versus PT: Additional results

Table S2 shows tests of the predictive performance of the NCM model versus PT on the data of L'Haridon and Vieider (2019). Given that there are 30 countries, and that I conduct the test separately on gains and losses, this gives me 60 tests, which are independent conditional on the experimental design. The NCM clearly outperforms PT in all 60 cases.

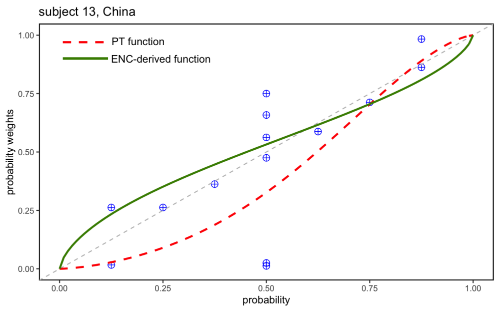
I illustrate the model-fit using some additional individual-level fit plots. I focus on

Table S2: Model fit of NCM versus PT in L'Haridon and Vieider (2019)

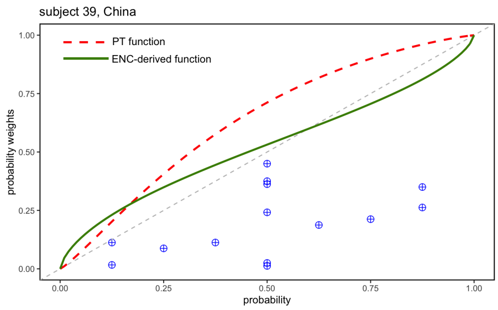
	LEPD difference			LEPD difference	
country	gains	losses	country	gains	losses
Australia	−2374.5	−2147.3	Kyrgyzstan	−3783.6	−3425.8
(SE diff)	(13.3)	(12.1)	(SE diff)	(16.0)	(14.3)
Belgium	−3549.8	−3198.2	Malaysia	−2493.7	−2257.5
(SE diff)	(15.4)	(15.3)	(SE diff)	(13.5)	(12.1)
Brazil	−3277.0	−2949.3	Nicaragua	−4680.6	−4236.3
(SE diff)	(14.8)	(14.7)	(SE diff)	(17.7)	(16.2)
Cambodia	−3116.3	−2821.8	Nigeria	−7875.2	−7088.4
(SE diff)	(15.1)	(13.5)	(SE diff)	(23.1)	(20.7)
Chile	−3745.0	3361.8	Peru	−3705.2	−3346.2
(SE diff)	(15.8)	(16.7)	(SE diff)	(15.7)	(15.1)
China	−7952.8	−7191.2	Poland	−3468.2	−3128.5
(SE diff)	(23.2)	(21.6)	(SE diff)	(15.4)	(15.1)
Colombia	−4299.8	−3806.5	Russia	−2715.6	−2445.9
(SE diff)	(18.2)	(18.1)	(SE diff)	(15.3)	(14.9)
Costa Rica	−4134.9	−3725.4	Saudi Arabia	−2535.8	−2280.4
(SE diff)	(16.6)	(17.1)	(SE diff)	(18.0)	(13.3)
Czech Rep.	−3857.2	−3479.0	South Africa	−2769.4	−2499.1
(SE diff)	(16.2)	(15.7)	(SE diff)	(13.7)	(13.2)
Ethiopia	−5457.3	−4882.1	Spain	−3121.1	−2817.2
(SE diff)	(19.5)	(19.2)	(SE diff)	(14.4)	(13.9)
France	−3619.3	−3257.9	Thailand	−3078.2	−2764.3
(SE diff)	(16.5)	(16.4)	(SE diff)	(14.6)	(15.7)
Germany	−5067.6	−4574.1	Tunisia	−2872.4	−2603.4
(SE diff)	(18.4)	(18.1)	(SE diff)	(15.5)	(13.1)
Guatemala	−3264.9	−2958.4	UK	−3112.5	−2818.2
(SE diff)	(16.2)	(14.3)	(SE diff)	(15.2)	(13.7)
India	−3468.6	−3138.0	USA	−3782.7	−3385.5
(SE diff)	(15.7)	(14.4)	(SE diff)	(16.0)	(14.8)
Japan	−3274.1	−2956.4	Vietnam	−3387.6	−3064.2
(SE diff)	(14.8)	(14.4)	(SE diff)	(15.9)	(14.5)

The reported LEPD differences (log estimated predicted density differences) indicate the comparative performance of the two models (Gelman et al., 2014a; Vehtari et al., 2017). Negative differences constitute evidence in favour of the NCM over PT. All tests are significant at $p < 0.001$. Tests using WAIC yield similar results, and are thus not reported for parsimony.

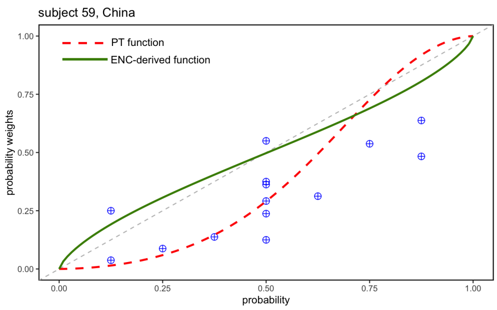
individuals with $\widehat{\gamma} > 1.1$ in the data of L'Haridon and Vieider (2019), which are cases that are at first approximation difficult to account for by the NCM, which imposes $\gamma < 1$. Oftentimes, these are the observations that will have $\widehat{\alpha} \gg 1$ as well, as shown in the correlation graph in the main text. Rather than randomly selecting subjects, below I start by documenting the patterns in order for each individual displaying the described pattern, proceeding in order by subject number for the same 4 countries used as examples in the main text, unless stated otherwise.



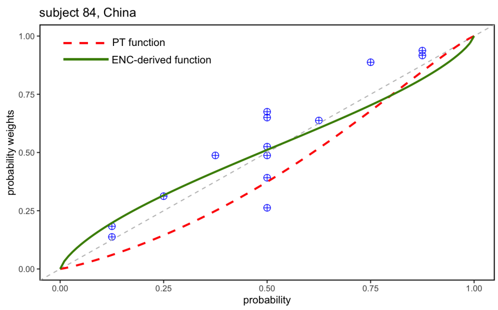
(a) Subject 13



(b) Subject 39



(c) Subject 59

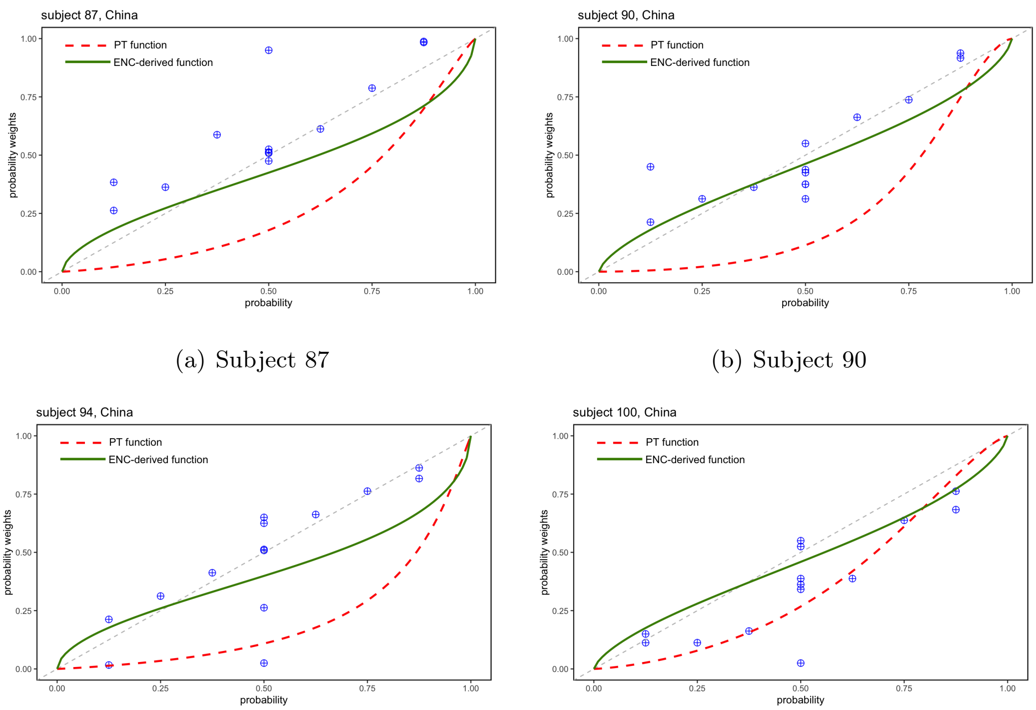


(d) Subject 84

Figure S22: Individual-level data fit of PT vs NCM in L'Haridon and Vieider (2019)

Figure S22 shows the fit for the first 4 individuals. Th pattern in panel 22(a) shows a pattern very similar to the one seen in the main text. Under PT, this subject displays oversensitivity at $\widehat{\gamma} = 1.45$, oversensitivity to outcomes at $\widehat{\alpha} = 2.39$, and low optimism at $\widehat{\delta} = 0.49$, as well as moderately high levels of noise, at $\widehat{\omega} = 0.14$. The NCM-derived parameters, which provide a clearly better fit at least to the probability dimension, come in at $\gamma = 0.68$, $\alpha = 0.74$, and $\delta = 1.14$. Optimism in probability-distortions is thus substituted for the extreme distortion of outcomes, obtained under PT from relatively few observations.

The other comparisons show similarly clear evidence in favour of the NCM-derived functions over PT. The fit in panel 22(b) appears poor for both functions, but this derived from the relatively high levels of outcome-distortion under both models ($\alpha = 0.64$, with a virtually identical parameter under PT). Beyond this, the NCM-derived function seems to run at least roughly parallel to the data points, whereas the PT-parameter $\widehat{\gamma} = 1.17$ clearly over-estimates probabilistic sensitivity. A similar observation holds for 22(c), where the PT parameters would appear to fit the evidence better for $p \leq 0.5$, but where the strong S-shape also means that the fit for large probabilities is very poor, pointing at a slightly different manifestation of overfitting. By reproducing the qualitative patterns seen in the displayed probability patterns, and absorbing the level difference through outcome-distortions, the NCM-model again outperforms PT. In panel 22(d), the patter is crystal clear, and needs no further discussion. Figure S23 shows similar patterns for the next 4 subjects.



(a) Subject 87

(b) Subject 90

(c) Subject 94

(d) Subject 100

Figure S23: Individual-level data fit of PT vs NCM in L'Haridon and Vieider (2019)

S3 Separability violations under ambiguity

S3.1 Experimental details

Subjects. 48 subjects were recruited at the Melessa Lab at the University of Munich in June 2011. Only subjects who had participated in less than 3 experiments previously were invited. One subject was eliminated because she manifestly did not understand the task, and alternately chose only the sure amount or only the prospect. 38% of the subjects were male and the average age was 25 years. The experiment was run using paper and pencil.

Experimental tasks. I presented subjects with 56 different binary prospects (28 for gains, 26 for losses, and 2 mixed prospects over gains and losses). Subjects had to make a choice between these prospects and different sure amounts of money, bounded between the highest and the lowest amount in the prospect. Gains were always presented first, and losses were administered from an endowment in a second part, the instructions for which were distributed once the first part was finished. Prospects were always kept in a fixed order. A pilot showed that this made the task less confusing for subjects, while no significant differences were found in certainty equivalents for different orders. Table S3 shows the prospects used in the usual notation $(x, p; y)$, where p indicates the probability of winning or losing x, and y obtains with a complementary probability $1 - p$.

Table S3: Decision tasks under risk and ambiguity

risky gains	**uncertain gains**	**risky losses**	**uncertain losses**
{0.5: 5; 0}	{0.5: 5; 0}	{0.5: –5; 0}	{0.5: –5; 0}
{0.5: 10; 0}	{0.5: 10; 0}	{0.5: –10; 0}	{0.5: –10; 0}
{0.5: 20; 0}	{0.5: 20; 0}	{0.5: –20; 0}	{0.5: –20; 0}
{0.5: 30; 0}	{0.5: 30; 0}	{0.5: –20; –5}	{0.5: –20; –5}
{0.5: 30; 10}	{0.5: 30; 10}	{0.5: –20; –10}	{0.5: –20; –10}
{0.5: 30; 20}	{0.5: 30; 20}	{0.125: – 20; 0}	{0.125: – 20; 0}
{0.125: 20; 0}	{0.125: 20; 0}	{0.125: –20; –10}	{0.125: –20; –10}
{0.125: 20; 10}	{0.125: 20; 10}	{0.25: –20; 0}	{0.25: –20; 0}
{0.25: 20; 0}	{0.25: 20; 0}	{0.385: –20; 0}	{0.385: –20; 0}
{0.385: 20; 0}	{0.385: 20; 0}	{0.625: –20; 0}	{0.625: –20; 0}
{0.625: 20; 0}	{0.625: 20; 0}	{0.75: –20; 0}	{0.75: –20; 0}
{0.75: 20; 0}	{0.75: 20; 0}	{0.875: –20; 0}	{0.875: –20; 0}
{0.875: 20; 0}	{0.875: 20; 0}	{0.875: –20; –10}	{0.875: –20; –10}
{0.875: 20; 10}	{0.875: 20; 10}	**mixed:** {0.5: 20; **–L**}	**mixed:** {0.5: 20; **–L**}

Prospects are displayed in the format $(p : x; y)$.

Notice how the exact same prospects were administered for risk (known probabilities) and uncertainty (unknown or vague probabilities). This will allow me to study ambiguity attitudes, i.e. the difference in behavior between uncertainty and risk. Preferences were elicited using choice lists, with sure amounts changing in equal steps between the extremes of the prospect.

Incentives. At the end of the game, one of the tasks was chosen for real play, and then one of the lines for which a choice had to be made in that task. This provides an incentive to reveal one's true valuation of a prospect, and is the standard way of incentivizing this sort of task. Subjects obtained a show-up fee of €4. The expected payoff for one hour of experiment was above €15.

Risk and uncertainty. Risk was implemented using an urn with 8 consecutively numbered balls. Uncertainty was also implemented using an urn with 8 balls, except that subjects were now told that, while the balls all had a number between 1 and 8, it was possible that some balls may recur repeatedly while others could be absent. The description as well as the visual display of the urns closely followed the design of Abdellaoui et al. (2011). The main differences were that I ran the experiment using paper and pencil instead of with computers; that I used numbers instead of colours in order to allow for black and white printing; and that I ran the experiment in sessions of 15-25 subjects instead of individually.

S3.2 The decision model and estimation code

The decision model closely follows the one for risk used in section **??**, but with all parameters doubled for ambiguity. Ambiguity parameters are coded as deviations of the equivalent parameters for risk, mostly to increase computational efficiency and without loss of generality. All estimations are executed directly using the density around the switching point.