# Take your Time or Take Your Chance: Time Discounting as a Distorted Probability \*

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## Abstract

In an experimental setup explicitly invoking the intertwined nature of risk and time, we convert the time delay with which an outcome is received into a subjective probability to immediately obtain that same outcome. Under the standard model of discounted expected utility (DEU), such a probability simply measures time discounting. In contrast to the prediction of DEU, we observe however that time discounting thus obtained systematically differs from discounting based on risk-free tradeoffs of outcomes. We then show that, once we adopt a behavioral generalization of DEU, the aforementioned time discounting results from a distorted probability, where the distortion has two main sources. The first comes from the difference between utility for risk and intertemporal substitution. The second stems from non-linear probability weighting. Once these two behavioral anomalies are factored out, discounting becomes invariant to the presence or absence of risk in intertemporal tradeoffs of outcomes.

**Keywords:** time preferences; risk preferences; preference modeling; discounted expected utility; behavioral economics

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## 1. Introduction

Discounted Expected Utility (DEU) — the workhorse of modern economics when it comes to the modeling of savings and consumption in the presence of risk (Becker and Mulligan, 1997; Yaari, 1965; Zeldes, 1989) — results from the combination of two separate lines of inquiry regarding individual decision making. The first suggests to evaluate deterministic streams of outcomes by a sum of utilities of the outcomes weighted by exponentially decreasing discount factors, hence resulting in the discounted utility model (DU; Samuelson, 1937). The second line of inquiry focuses on a temporal decision making under uncertainty and has resulted in the popular expected utility model (EU; Savage, 1954; von Neumann and Morgenstern, 1944). Under DEU, a risky prospect over deterministic streams of outcomes is evaluated as the expectation of the discounted utilities of those streams. Whether a choice involves risky intertemporal tradeoffs or not, DEU uses one and the same discount function and utility index to evaluate objects of choice. The descriptive validity of DEU has, however, been challenged by recent experimental findings showing that both utility (Abdellaoui et al., 2013; Andreoni and Sprenger, 2012a; Brown and Kim, 2014) and time discounting (Andreoni and Sprenger, 2012b) may depend on whether the choices used to elicit them are risky or riskless.<sup>1</sup>

The present paper proposes an experimental investigation of discounting in a setup that explicitly invokes the intertwined nature of risk and time. Specifically, our setup allows us to convert the delay with which an outcome is received into the subjective probability to immediately get that same outcome, which we refer to as the risk equivalent of the delay. Under DEU, this probability turns out to be a direct, nonparametric measure of discounting. It can further be used to illustrate the impact of two empirically robust choice anomalies on discounting when intertemporal tradeoffs are risky. The first anomaly comes from the observed difference between utility for risk and utility for time (Abdellaoui et al., 2013; Andreoni and Sprenger, 2012a). The second is related to the systematic tendency of people to transform probabilities in a non-linear fashion (Barseghyan et al., 2013; Bruhin et al., 2010; Wakker, 2010). Assuming a descritpively motivated extension of DEU that accounts for the two aforementioned choice anomalies suggests that discounting measured through risk equivalents should be thought of as a distorted probability. Correcting for the relevant distortions should thus result in the invariance of discounting to the presence or

<sup>&</sup>lt;sup>1</sup>Formally, in the present paper we opt for a simplified version of DEU where outcomes are directly assigned an objective probability instead of probabilities resulting from an explicitly given history of events. In our setup no delayed resolution of uncertainty occurs, which is in contrast to Kreps and Porteus' (1978) recursive expected utility.

absence of risk in intertemporal tradeoffs of outcomes.

Let us illustrate this proposition. Assume that an agent is indifferent between obtaining \$200 in six months' time, represented as (x, t), and the prospect that gives her the same \$200 immediately with an 80% probability, represented as (x, p).<sup>2</sup> Under DEU, this indifference implies that the value of the temporal prospect, D(t)u(x), where D and u stand for the discount and utility function respectively, will be equal to the value of the risky prospect, pu(x). Because DEU uses a single utility index for both time and risk, we can cancel out the term u(x) and obtain

$$D(t) = p. (1)$$

This means that one can fix a time t along with an outcome x and directly obtain the corresponding discount factor from the risk equivalent p.<sup>3</sup> In terms of the previous example, the agent applies a discount factor of D(t) = 0.8 when the receipt of \$200 is delayed by six months. This approach mimics that of Anscombe and Aumann's (1963) subjective EU.<sup>4</sup> By introducing initial delays, the method is furthermore flexible enough to obtain non-parametric estimates of deviations from constant discounting (see also Rohde, 2010).

Notice how this setup allows us to non-parametrically measure discounting under DEU without having to worry about measuring utility. Traditional measurements of discounting have often assumed linear utility, which may bias the elicited function if utility is truly nonlinear (Frederick et al., 2002, p. 381). Our method allows us to access the discount function directly and without the utility confound when assuming DEU.<sup>5</sup> It furthermore can also be used to illustrate in an intuitive fashion the potential biases in estimated discounting that could result from domain-specific utility functions for risk and time, and from nonlinear probability weighting.

Recently reported violations of DEU suggesting the existence of specific utility indices for risk and for time indeed cast doubt on the capacity of Eq. (1) to elicit discounting in an unbiased fashion. In particular, they show that, contrary to the assumption underlying DEU, intertemporal utility differs from risky utility. Assume now a specific utility scale,

<sup>&</sup>lt;sup>2</sup>Within our "time-risk" setup, (x,t) can be identified with the lottery giving (x,t) with certainty. Similarly, lottery (x, p) consists of the lottery giving x at t = 0 with probability p and nothing otherwise.

<sup>&</sup>lt;sup>3</sup>Assuming EU for risk and DU for temporal prospects, Takeuchi (2011) uses a two-step procedure to elicit discounting that results in Eq. (1).

<sup>&</sup>lt;sup>4</sup>In particular, the availability of objective probabilities makes it possible to assign subjective probabilities to uncertain events through matching probabilities. The subjective probability of an uncertain event E is thus determined by finding the matching probability p that makes the agent indifferent between (x, p)and the gamble that gives x when E occurs and nothing otherwise, i.e. (x, E). This allowed Anscombe and Aumann (1963) to bridge risk and uncertainty under EU.

<sup>&</sup>lt;sup>5</sup>Another method to elicit discounting without the interference of utility was recently proposed by Attema et al. (2016) within a DU framework.

v, for intertemporal tradeoffs between deterministic outcomes, and a different utility scale, u, for risky tradeoffs. Having two utility scales over the same set of outcomes means that there necessarily exists a strictly increasing function  $\phi$  that transforms the intertemporal utility scale v into the risky utility scale u, i.e.  $u = \phi \circ v$ . Since the value of (x, t) is measured on the scale v, one needs to convert it into the scale u when evaluating the aforementioned indifference between (x, t) and (x, p), which involves both time delays and risk. The new equation thus becomes  $\phi[D(t)v(x)] = pu(x)$ . After normalizing both utility scales<sup>6</sup>, we obtain

$$D(t) = \phi^{-1}(p),$$
 (2)

where  $\phi^{-1}$  stands for the inverse of  $\phi$ . This shows that the existence of domain-specific utility functions necessarily impacts discounting as well. In other words, one would expect that discounting elicited in risky environments is not the same as discounting obtained from riskless tradeoffs if the utilities for time and risk are indeed different.

Even once one allows for different utility under risk and over time, another potential source of bias in the measurement of discounting results from the long list of observations showing that most people transform probabilities in a nonlinear fashion (Barseghyan et al., 2013; Bruhin et al., 2010; Tversky and Kahneman, 1992). Allowing for a probability weighting function, w, that is used to transform probabilities into decision weights, Eq. (2) becomes

$$D(t) = \phi^{-1}[w(p)].$$
 (3)

This equation shows how nonlinear probability weighting may impact discounting when risk and time are simultaneously at play (see also Epper and Fehr-Duda, 2015). More specifically, the use of the risk equivalent method in a setting where agents assign domaindependent utilities to outcomes and transform probabilities nonlinearly results in discount factors that are impacted by both  $\phi$  and w. Eq. (3) then points to a generalization of DEU that combines a domain-dependent utility specification with rank-dependent utility for risk. Correcting discounting for potential biases arising from domain-dependent utility and/or non-linear probability weighting thus requires an extension of DEU that accounts for the two new components  $\phi$  and w. The resulting model presented in Section 2 has enough flexibility to relate discounting to a distorted risk equivalent as in Eq. (3). Note that, although a generalization of DEU like that proposed by Baucells and Heukamp (2012) also accounts for non-linear probability weighting in an intertemporal setting, the latter

<sup>&</sup>lt;sup>6</sup>Because u and v are unique only up to a positive linear transformation, one can use the normalization conditions u(x) = v(x) = 1 for a fixed outcome x.

component cancels out when risk equivalents are used. In other words, their model results in the same (constant) discounting as predicted by DEU for risk equivalents. Additionally, the model of Baucells and Heukamp cannot account for the difference between utility for risk and intertemporal substitution.<sup>7</sup>

The descriptive power of our generalized DEU (henceforth GDEU) model can thus be evaluated simply by checking whether allowing for domain-specific utilities along with nonlinear probability weighting results in a convergence between the discount function obtained in a riskless environment and that obtained from risk equivalents. We can also investigate whether accounting for one single choice anomaly (i.e., either  $\phi$  or w) is sufficient to obtain an unbiased discount function, and thus to identify the most parsimonious model allowing for the unbiased measurement of discounting.

To test our model, we thus experimentally measure discounting under risk and in riskless environments. When applying DEU, we find discount factors to differ significantly between the two setups and to do so systematically, with more pronounced discounting in riskless environments than under risk. This points to the importance of risk for the measurement of time discounting, as predicted by our model. We find risky utility to be more concave than inter-temporal utility, confirming previous findings and showing the importance of modeling this difference explicitly. We then proceed to correcting the discount factors obtained under risk by adjusting them for the difference in utilities  $\phi$  and for probability weighting w, as postulated in our model. The correction closes the gap between risky and riskless discounting, thus empirically validating our modeling approach. We also show that the correction for different utility curvatures in itself is not sufficient to obtain this result, although it somewhat narrows the gap between the two discount functions. We thus conclude that while risky utility is domain-specific, discounting is not—the same discounting function obtains in risky and riskless setups once risk preferences are taken into account in their full richness.

The present paper proceeds as follows. Section 2 introduces the theory and measurement approach. Section 3 presents the results in four subsections: 3.1 presents nonparametric results derived directly from risk equivalents based on DEU; 3.2 compares discount factors obtained from risk equivalents under DEU to discount factors measured in a riskless environment (DU); 3.3 compares risky and riskless utility; 3.4 applies correction factors derived from our model to discounting under risk and compares the corrected discount

<sup>&</sup>lt;sup>7</sup>Under Baucells and Heukamp's 2012 model, using risk equivalents, i.e., indifferences  $(x, t) \sim (x, p)$ , results in Eq. (1). Equations (2) and (3) require a more general formal setup (see also Abdellaoui et al., 2017).

factors thus obtained to the riskless discount factors; and section 3.5 discusses discounting patterns. Finally, section 4 discusses the results and concludes the paper.

## 2. Theory and measurement

The present section is divided into four subsections. We start by presenting our general model and its connections to other modeling approaches. We then present a simplified version of the model and discuss our strategy for the identification of the model parameters. We continue by presenting the elicitation methods used to determine the different parameters of the model. The econometric estimation approach is explained in the Appendix.

## 2.1. A unified time-risk preference framework

We consider agents facing choices involving risk and time. Consequences of choices are streams of monetary outcomes  $\boldsymbol{x} = (x_0, x_1, ..., x_T)$ , where  $x_t$  is received at time t and Tindicates a fixed time horizon. Objects of choice, called *risk-time prospects*, are probability distributions over streams of outcomes, i.e.  $\boldsymbol{p} = (p_1, \boldsymbol{x_1}; ...; p_n, \boldsymbol{x_n})$ , where  $\boldsymbol{x_i} = (x_{i0}, ..., x_{iT})$ . Uncertainty is always resolved immediately in our setup. For the sake of simplicity, a prospect giving  $\boldsymbol{x}$  for sure is also denoted by  $\boldsymbol{x}$ . The set of objects of choice is endowed with a preference relation  $\succeq$ , with  $\sim$  and  $\succ$  defined as usual. This relation is assumed to be transitive and complete. In the sequel we assume without loss of generality that  $\boldsymbol{x_1} \succeq ... \succcurlyeq \boldsymbol{x_n}$ .

Under DEU, a risk-time prospect p is assigned the value

$$DEU(\mathbf{p}) = \sum_{i=1}^{n} p_i \sum_{t=0}^{T} D(t)u(x_{it}),$$
(4)

where u is a strictly increasing utility function defined from  $\mathbb{R}$  to  $\mathbb{R}$ , which is unique up to a positive linear transformation, and D represents a discount function, i.e. a strictly decreasing function from nonnegative numbers to the unit interval, with D(0) = 1. Note that Eq. (1),  $D(t) = \phi^{-1}(p)$ , results from the application of Eq. (4) to the indifference  $(x, p) \sim (x, t)$ .

We now amend the model given in Eq. (4) to allow for domain-specific utility and nonlinear probability weighting. We call this model generalized discounted expected utility (GDEU):

$$GDEU(\boldsymbol{p}) = \sum_{i=1}^{n} \pi_i \phi \left[ \sum_{t=0}^{T} D(t) v(x_{it}) \right],$$
(5)

where  $\pi_i$  is a decision weight replacing the probability  $p_i$ , which is defined as follows:  $\pi_i = w(\sum_{j=1}^i p_j) - w(\sum_{j=1}^{i-1} p_j), i = 1, ..., n$ . The weighting function w is a strictly increasing function on the unit interval satisfying w(0) = 0 and w(1) = 1. The component v stands for a utility function defined over temporal outcomes. The function  $\phi$  is defined from  $v(\mathbb{R})$  to  $\mathbb{R}$ , and is a strictly increasing and continuous transformation function. For fixed utility v the transformation  $\phi$  is defined up to an increasing linear transformation. Additionally, the utility v can be replaced by av + b if  $\phi(.)$  is replaced by  $\phi((.-b)/a), a > 0, b \in \mathbb{R}$ .

We now discuss three different subcases of this model—DU, rank-dependent utility (RDU), and DEU with a domain-dependent utility. We start from DU. Assume that z (i.e. the stream  $z_0\mathbf{0}$ ) is the present equivalent of a stream of outcomes  $\boldsymbol{x} = (x_0, ..., x_T)$ , i.e. the amount paid out at time 0 that the agent considers equally good as obtaining the stream  $(x_0, ..., x_T)$ . Further assume without loss of generality that v(0) = 0. Under GDEU  $\phi[v(z)]$  is equal to  $\phi\left[\sum_{t=0}^{T} D(t)v(x_t)\right]$ . Given that  $\phi$  is strictly increasing this simplifies to

$$v(z) = \sum_{t=0}^{T} D(t)v(x_t),$$
(6)

which is DU with v as a utility function. In our model v thus captures the utility of riskless temporal outcomes.

Let us now consider the RDU restriction of our model. Assume a risk-time prospect  $\boldsymbol{p}$  such that  $\boldsymbol{p} = (p_1, \boldsymbol{x_1}; ...; p_n, \boldsymbol{x_n})$ , where  $\boldsymbol{x_i} = (x_{i0}, 0, ..., 0)$ , i = 1, ..., n. That is, only the outcome received at t = 0 may be different from 0. Assuming again v(0) = 0 we obtain  $GDEU(\boldsymbol{p}) = \sum_{i=1}^{n} \pi_i \phi [v(x_{i0})]$ . Defining utility for risk as  $\phi \circ v$ , the equation above results in the standard RDU model.

Finally, we consider a version of our model where linearity of utility in the probabilities holds, but  $\phi$  is not necessarily the identity function. This results in

$$GDEU(\boldsymbol{p}) = \sum_{i=1}^{n} p_i \phi \left[ \sum_{t=0}^{T} D(t) v(x_{it}) \right].$$
(7)

This equation shows that accounting for the discrepancy between riskless intertemporal

utility and risky utility through  $\phi$  necessarily impacts both utility and discounting.

In the absence of nonlinear probability weighting our preference functional is formally similar to the recursive model initially proposed by Kreps and Porteus (1978) and the preference functional used in the smooth ambiguity model by Klibanoff et al. (2005). In Kreps and Porteus (1978), the  $\phi$  reflects delayed resolution of uncertainty, while in Klibanoff et al. (2005) it reflects ambiguity attitudes. In our model, where no delayed resolution of uncertainty is at play,  $\phi$  reflects the impact of immediately resolved risk on time preferences.

## 2.2. Model component identification in a simplified setup

To identify the parameters of our model, we elicit three quantities in a simplified choice context: risk equivalents (*REs*), certainty equivalents (*CEs*), and time equivalents (*TEs*). We restrict ourselves to binary prospects defined over streams of at most two outcomes. This requires simplified notation. If the agent receives a single future outcome at some date t, in the general setup the resulting stream can be written as  $(0, ..., x_t, ..., 0)$ , for which we will use the shorthand  $x_t$ . By the same token, a notation  $y_s x_\ell$  represents a stream of two outcomes where x is received sooner after a delay s and y is received later after a delay  $\ell$ , and all other outcomes of the stream are equal to 0. A risk-time prospect is then defined as  $(x_s x'_\ell, p; y_s y'_\ell)$ , where the stream  $x_s x'_\ell$  obtains with a probability p, and the the stream  $y_s y'_\ell$  obtains with a complementary probability 1 - p.

#### **Risk** equivalents

A RE corresponds to a probability that makes an agent indifferent between the risky prospect,  $(x_s, p; 0)$ , that gives x with probability p or else 0 at time s, and the temporal prospect  $(x_\ell, 1; 0)$ , which gives the *same* outcome x for sure at time  $\ell$ . In the introduction, we considered the special case in which s = 0 for illustrative purposes. Under GDEU and after normalizing utility of the given outcome x, i.e. v(x) = 1, this indifference implies<sup>8</sup>

$$w(p)\phi[D(s)] = \phi[D(\ell)]. \tag{8}$$

<sup>&</sup>lt;sup>8</sup>The general equation is  $w(p)\phi[D(s)v(x)] = \phi[D(\ell)v(x)]$ . Since we use a fixed outcome x (except for a consistency check not included in the main estimations), we can normalize the utility function. A parametric specification of  $\phi$  as a power function, as used in the present paper, leads to the same result without a need for normalization.

Note that in case s = 0 and  $\ell > 0$ , as in the example of the introduction, this equation simplifies to  $D(\ell) = \phi^{-1}[w(p)]$  under GDEU and to  $D(\ell) = p$  under DEU. While the latter equation can be fully identified from REs, the general Eq. (8) requires  $\phi$  and w to be identified. This is achieved by eliciting CEs and TEs.

## Certainty equivalents

A CE consists of a sure amount  $c_0$  that the agent considers equally good as a two-outcome prospect  $(x_0, p; y_0)$ , where x > y. Under GDEU, this results in

$$\phi[v(c)] = w(p)\phi[v(x)] + (1 - w(p))\phi[v(y)].$$
(9)

By obtaining such indifferences for different values of p and different outcomes x and y, we can identify both w and  $u = \phi \circ v$ . To separately identify  $\phi$  and v we use TEs.

## Time equivalents

Assume three outcomes y, y', and x. A TE received at time s is the outcome z such that  $z_s y'_{\ell} \sim y_s x_{\ell}$ . If s = 0 and y' = 0, the time equivalent is called a present equivalent. In other words, the PE of a stream of monetary outcomes,  $y_s x_{\ell}$ , is defined as the outcome  $z \in \mathbb{R}$  such that  $z_0 \sim y_s x_{\ell}$ . That is, z is the outcome paid immediately (at t = 0) that the agent considers equally good as obtaining the stream of outcomes  $y_s x_{\ell}$ . In terms of GDEU, this indifference implies

$$D(s)v(z) + D(\ell)v(y') = D(s)v(y) + D(\ell)v(x).$$
(10)

This equation allows us to identify v and D under DU. Regarding the latter, taking this equation separately allows us to empirically test whether discounting obtained under risk using REs is the same as discounting obtained trading off riskless outcomes. Having identified v, we can now also identify  $\phi$ .

## 2.3. Experimental procedures and stimuli

We recruited 104 subjects (41% female) at the experimental laboratory of the Technical University Berlin, Germany, using ORSEE (Greiner, 2004). The experiments were run in March and April 2016 and were conducted in individual interviews by four experimenters. On average, the interviews lasted one hour. Upon arrival, subjects were invited to sit down in front of a computer. They were then shown a recorded video presenting the experimental instructions (available upon request). They were also given written instructions, included in the online appendix (see Online Appendix B). After watching the video, reading the instructions, and asking any questions they still had, they were presented with five comprehension questions. Before starting the experiment, they furthermore answered two practice questions, during which the experimenter further clarified any remaining doubts.

Stimuli were grouped by type to avoid confusion, and both the order of blocks of stimuli (REs, CEs, TEs) and of stimuli within each block was randomized. The complete stimuli are shown in table 1. The headings of the table indicate the tradeoffs faced in our simplified notation. The stimuli are then described in the format of the headers, with an asterisk \* marking the dimension varying within the choice list. One stimulus of each type was repeated at a randomly selected moment, which allows us to test for the consistency of responses and helps identifying the error term in econometric estimations.

		REs				CEs		TEs	
NR	$(x_s, p; 0)$	$\sim$	$(x_\ell, 1; 0)$	$c_0$	$\sim$	$(x_0, p; y_0)$	$z_s x'_\ell$	$\sim$	$y_s x_\ell$
1	$(30_0, p^*; 0)$	$\sim$	$(30_1, 1; 0)$	$c_0^*$	$\sim$	$(10_0, 0.5; 0_0)$	$z_0^* 0_1$	$\sim$	$0_0 3 0_1$
2	$(30_0, p^*; 0)$	$\sim$	$(30_3, 1; 0)$	$c_0^* c_0^* c_0^$	$\sim$	$(20_0, 0.5; 0_0)$	$z_0^*0_3$	$\sim$	$0_0 3 0_3$
3	$(30_0, p^*; 0)$	$\sim$	$(30_6, 1; 0)$	$c_0^{*}$	$\sim$	$(30_0, 0.5; 0_0)$	$z_{0}^{*}0_{6}$	$\sim$	$0_0 3 0_6$
4	$(30_0, p^*; 0)$	$\sim$	$(30_9, 1; 0)$	$c_0^*$	$\sim$	$(30_0, 0.5; 10_0)$	$z_{0}^{*}0_{9}$	$\sim$	$0_0 3 0_9$
5	$(30_0, p^*; 0)$	$\sim$	$(30_{12}, 1; 0)$	$c_0^*$	$\sim$	$(25_0, 0.5; 5_0)$	$z_0^* 0_{12}$	$\sim$	$0_0 30_{12}$
6	$(30_3, p^*; 0)$	$\sim$	$(30_6, 1; 0)$	$c_0^*$	$\sim$	$(30_0, 0.1; 0_0)$	$z_{3}^{*}0_{6}$	$\sim$	$0_{3}30_{6}$
7	$(30_6, p^*; 0)$	$\sim$	$(30_9, 1; 0)$	$c_0^{\check{*}}$	$\sim$	$(30_0, 0.3; 0_0)$	$z_{6}^{*}0_{9}$	$\sim$	$0_{6}30_{9}$
8	$(30_9, p^*; 0)$	$\sim$	$(30_{12}, 1; 0)$	$c_0^{*}$	$\sim$	$(30_0, 0.7; 0_0)$	$z_{9}^{*}0_{12}$	$\sim$	$0_9 3 0_{12}$
9	$(30_{11}, p^*; 0)$	$\sim$	$(30_{12}, 1; 0)$	$c_0^*$	$\sim$	$(30_0, 0.9; 0_0)$	$z_{11}^*0_{12}$	$\sim$	$0_{11}20_{12}$
10	$(20_0, p^*; 0)$	$\sim$	$(20_6, 1; 0)$	$c_0^*$	$\sim$	$(30_0, 0.95; 0_0)$	$\bar{z_{0}^{*}}0_{6}$	$\sim$	$0_0 20_6$
11	$(20_0, p^*; 0)$	$\sim$	$(20_6, 1; 0)$	Ŷ			$z_0^*10_6$	$\sim$	$10_0 30_6$
12	,						$z_0^* 20_6$	$\sim$	$0_0 3 0_6$
13							$z_0^{*}20_6$	$\sim$	$10_0 30_6$
14							$z_0^* 20_6$	$\sim$	$20_0 30_6$

Table 1: Experimental stimuli

NOTE: All outcomes are in euros. At the time of the experiment,  $@1 \simeq \$1.20$ ; time delays are indicated in months

Subjects faced a total of 11 unique RE tasks. All but two (NRs 10 and 11), used for a consistency check, offered a fixed outcome of  $\in 30$ . The REs varied either the sooner

time s or the later time  $\ell$ . The main difference of CEs from REs was that a sure outcome was varied instead of the probability. CEs could vary between the high amount x and low amount y of the risky prospect, and the time of payout was always fixed at s = 0. Half of the prospects kept outcomes fixed at  $\in$ 30 or else 0, and varied probabilities from 0.1 to 0.9. We used one additional probability of 0.95 to take account of choices skewed towards high probabilities in REs revealed in a pilot. In the second half of the risky prospects we kept the probability fixed at 0.5 while varying the outcomes x and y. For TEs we again followed similar procedures. Just as for CEs, we varied sure monetary amounts in a choice list, while time periods and other outcomes were fixed. The elicited amount was always paid out at the sooner period s. In 8 TEs we elicited indifferences between a sooner outcome and one later outcome fixed at  $\in$ 30. What varied between these lists were the sooner and later time period. In the remaining 6 TEs, the time periods were fixed at s = 0 and  $\ell = 6$ months. The prospects contained a richer outcome space to identify intertemporal utility. The outcome  $y_s$  was introduced so as to obtain natural limits for the choice lists on which  $z_0^*$  was measured.

Figure 1 shows a screenshot of a RE task. It represents a choice between a 20% probability of receiving  $\in$  30 in three months, or receiving  $\in$  30 for sure in 6 months. The probability of winning in the risky prospect ranged from 0 to 1 in steps of 0.01. In order to speed up the decision process, a bisection procedure was used to complete the list. Once the list had thus been completed, subjects were forced to check and validate the complete choice list. At this point, they could alter their choices if desired before confirming the list and moving on to the next task. Subjects were explicitly made aware that the bisection procedure was merely a decision aid, and that final payoffs would be determined by the complete underlying choice list. Similar displays were used for CEs and TEs (see instructions in Online Appendix B).

All subjects obtained a  $\in$ 5 show-up fee, which was paid in cash after the experiment. In addition, subjects were incentivized by paying one decision randomly selected from all the choices with equal probability—the standard procedure in this type of experiment. The extraction always took place directly after the experiment, and any uncertainty was resolved at that point. The performance-contingent payments happened exclusively by bank transfer to equalize transaction costs between sooner and later payments. Subjects were made aware that the wire transfer of the early outcome would be made the following

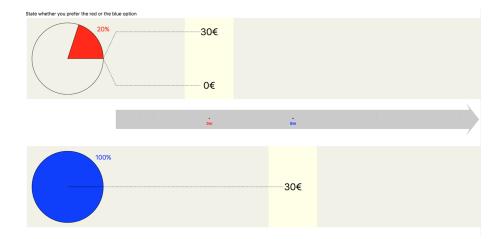


Figure 1: Screenshot of a RE task

day and that the money would arrive on their account three days after that. By the same token, any payout indicated at time  $\ell$  meant that the wire transfer would be postdated to that day, with the money arriving three days after the indicated time. The subjects obtained a certificate from the WZB Berlin Social Science Center, guaranteeing the delayed payoff. Subjects were familiar with the WZB, which maintains the experimental lab together with the Technical University. The certificate described the procedure, and contained information on the payment (amount and date of wire transfer), as well as contact numbers and emails of the administrators at the WZB making the transfer. Subjects were explicitly encouraged to contact the WZB in case they had any problems, or in case they were to change their bank account number before the payout date.

## 3. Results

We report the results in several steps. We start by reporting nonparametric data on discounting obtained from REs assuming DEU in section 3.1. In section 3.2 we compare discount factors obtained from REs under risk to riskless discount factors obtained from TEs, again under DEU. Section 3.3 zooms in on the utility dimension, and tests whether different utilities are needed under risk and over time. In section 3.4 we correct the discount factors obtained from REs for risk preferences (domain-dependent utility and nonlinear probability weighting), and compare the corrected discount factors to those obtained from TEs. Finally, in section 3.5 we test for deviations from exponential discounting.

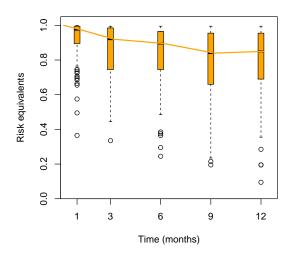


Figure 2: REs for time delays from s = 0

## 3.1. The discount function for risk-time prospects under DEU

We begin by describing the REs obtained for different delays from s = 0 assuming DEU. Figure 2 displays the discount factors together with their interquartile range. The resulting discount function slopes downward as one would expect. The implied annualized discount factors, however, are found to increase in the length of the delay (i.e., annualized discount rates are lower the longer the delay). For instance, the median annualized discount factor for a 3 months delay from the present is 0.72 (corresponding to an annual discount rate of 40%), while the discount factor for a one year delay is 0.85 (corresponding to an annual discount rate of 18%). Indeed, all annualized discount factors derived from shorter delays are significantly smaller than discount factors derived from longer delays (all pairwise differences are significant at p < 0.001, using a Wilcoxon signed-rank test).<sup>9</sup> This is consistent with earlier findings showing that lower discount rates are estimated for longer time delays (Read, 2001).

We next examine issues of stationarity, i.e. whether discounting is constant over time or follows different patterns such as decreasing (or potentially increasing) impatience. Table 2 shows pairwise comparisons of risk equivalents elicited for three months delays from

<sup>&</sup>lt;sup>9</sup>Two delays, of 6 months and 12 months from the present, where included using lower stakes of  $\in 20$  as a stability check. Using these two tasks, we find the same qualitative pattern of annualized discount factors increasing in the delay, with an annualized discount factor of 0.79 for the 6 months delay, and a discount factor of 0.85 for the 12 months delay (different at p < 0.001, signed-rank test).

different starting periods, indicated as  $p(s, \ell)$ . The table indicates the number of subjects with larger REs for the earlier period (increasing impatience), with equal REs for the two periods (constant impatience), and with smaller REs for the earlier period (decreasing impatience). Compared to the 3 months delay from s = 0 (shown in the first three data columns of the table), about one third of subjects can be seen to have constant discount rates, with this rate slightly declining in the initial delay of the comparison prospect. Of the remaining two thirds, a majority exhibits larger discount factors as the up front delay increases, indicating decreasing impatience. This asymmetry is highly significant as indicated by the Wilcoxon signed-rank tests reported in the last row of the table. Nonetheless, there is also a substantial minority of subjects (hovering around 20% of our sample) who exhibit *increasing* impatience (Attema et al., 2010).

Table 2: Classification of time inconsistencies

		p(0,3)		p(3	(, 6)	p(6,9)
versus	p(3,6)	p(6,9)	p(9, 12)	p(6,9)	p(9, 12)	p(9, 12)
$p(s,\ell) > p(s+k,\ell+k)$	23	19	22	25	26	29
$p(s,\ell) = p(s+k,\ell+k)$	38	34	29	40	37	43
$p(s,\ell) < p(s+k,\ell+k)$	43	51	53	39	41	32
signed-rank	z = -3.00	z = -4.07	z = -4.13	z = -1.63	z = -2.01	z = -0.40
test	p = 0.003	p < 0.001	p < 0.001	p = 0.104	p = 0.045	p = 0.686

These patterns tend to be much weakened when comparing the three months periods starting in 3 months or 6 months to later up front delays, as shown in the last three data columns of table 2. The number of time consistent subjects, indicating the same REs for different 3 months delays, is somewhat larger than in comparisons to 3 months delays from the present. And although there is still some asymmetry towards decreasing impatience, this pattern is now much weaker than in comparisons to delays from the present, with only the comparison of the 3 months delay starting in 3 months or in 9 months significant at conventional levels. This provides a first indication that generalized hyperbolicity is rather weak in our data at the aggregate level, while quasi-hyperbolic behavior tends to be stronger.

We further investigate this issue in figure 3, which shows the cumulative distributions for 3 months delays from different initial times. The CDF for the three months delay from s = 0 clearly stands out from the others, with people switching to the risky prospect at lower winning probabilities. The CDFs for other delays are much closer together, and differences are more difficult to spot, again suggesting a quasi-hyperbolic pattern rather

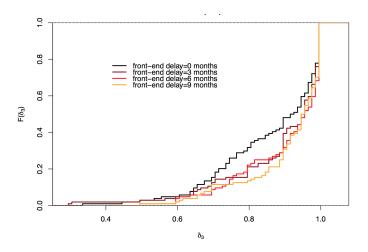


Figure 3: Cumulative distribution functions for 3-months delays

than a fully hyperbolic one in the aggregate data.

Finally, we can take a look at some typical patterns occurring at the individual level, shown in figure 4. These are derived by taking the ratio of later 3-month delays relative to a 3-months delay from the present, p(s,s+3)/p(0,3), where s = 3, 6, 9. With the initial discount factor normalized to 1 in the graph, changes in the index can then be taken as a direct measure of stationarity. In particular, a horizontal line indicates constant discounting, an upward sloping line decreasing impatience, and a downward sloping line increasing impatience.

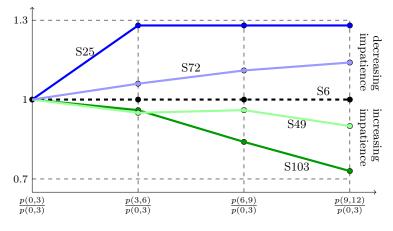


Figure 4: Hyperbolicity at the individual level: examples

The horizontal dashed line indicates constant discounting, which is exhibited by subject 6 and many others (as well as being the median pattern in the data). Subject 25 exhibits a quasi-hyperbolic pattern—there is decreasing impatience in the first segment, with stationarity prevailing thereafter. Subject 72 shows fully hyperbolic behavior, with a curve that slopes upward for each subsequent delay. Finally, subjects 49 and 103 show the opposite of hyperbolic behavior, i.e. they show increasing impatience. Impatience is strongly increasing for subject 103, with each subsequent comparison resulting in a increase in impatience. Subject 49 exhibits something closer to the opposite of quasi-hyperbolic behavior, with impatience registering in the first comparison and then staying approximately level. Clearly, these are but a few examples of behavior and other patterns exist, including more erratic ones.

## 3.2. Risky versus riskless discounting under DEU

We now compare discount factors derived from REs and involving risk to riskless discount factors derived from TEs. We assume DEU throughout this section—corrections of risky discounting for risk preferences in the generalized model will be discussed in section 3.4. Under the DEU assumption, there is indeed no reason to expect discounting to differ systematically between risky and riskless environments. Figure 5 shows the comparative discount factors for the different delay periods in our data derived from REs and TEs. The discount factors shown are based on maximum likelihood estimations which include intertemporal utility v for TEs to make them fully comparable to discount factors obtained from REs (i.e., the estimations of discount factors from TEs are based on Eq. (11)). Utility itself will be discussed in the next section.

It is easy to discern that the median discount factors obtained from riskless TEs are always smaller than those obtained from REs. In other words, discounting appears to be more severe when elicited under certainty than when elicited under risk. This provides a first indication that risk indeed has an impact on discounting, as postulated by our GDEU model. This in turn means that DEU is violated in our data. It also appears that the difference between risky and riskless discounting increases with the time delay, suggesting an effect on discount rates and not only on hyperbolicity.

Table 3 provides summary statistics of the different discount factors estimated under risk and under certainty and tests of their equality. The median discount factor can be seen to be lower in the riskless setup than in the risky one in all cases. This difference is indeed significant in all cases using either a Wilcoxon signed-rank test or a sign test on positive versus negative deviations. It remains statistically significant in all cases also when applying

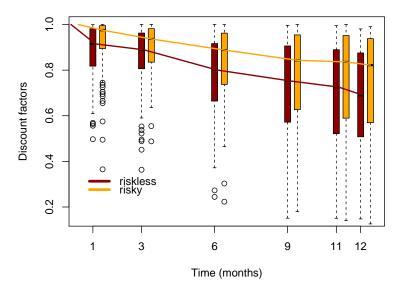


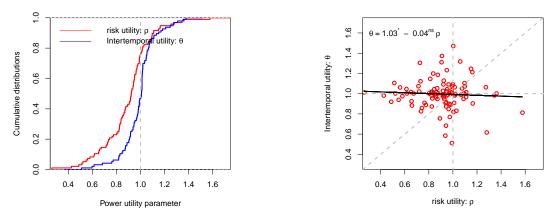
Figure 5: Discount factors for REs and TEs under DEU

	Risky		R	iskless	Comparison			
	Median	IQR	Median	IQR	Wilcoxon test	Sign test	Spearman Correlation	
$\frac{\delta_1}{\delta_3} \\ \delta_6 \\ \delta_9 \\ \delta_{11} \\ \delta_{12}$	$\begin{array}{c} 0.97 \\ 0.94 \\ 0.89 \\ 0.84 \\ 0.83 \\ 0.82 \end{array}$	$ \begin{bmatrix} 0.89 & ; & 0.99 \\ 0.84 & ; & 0.98 \\ 0.74 & ; & 0.96 \\ 0.63 & ; & 0.95 \\ 0.59 & ; & 0.95 \\ 0.57 & ; & 0.94 \end{bmatrix} $	$\begin{array}{c} 0.91 \\ 0.88 \\ 0.79 \\ 0.74 \\ 0.72 \\ 0.68 \end{array}$		-3.60*** -2.99** -4.64*** -4.01*** -3.79*** -3.93***	$74/30^{***}$ $66/38^{**}$ $74/30^{***}$ $74/30^{***}$ $74/30^{***}$ $74/30^{***}$	$\begin{array}{c} 0.36^{***} \\ 0.60^{***} \\ 0.69^{***} \\ 0.68^{***} \\ 0.62^{***} \\ 0.64^{***} \end{array}$	

Table 3: Discount factors under risk and under certainty assuming DEU

p-values in parentheses; all p-values reported are two-sided; \*: p < 0.05; \*\*: p < 0.01; \*\*\*: p < 0.001.

a Bonferroni adjustment, which requires p-values of 0.008 or smaller to account for the fact that the six tests are not independent of each-other. The difference is also economically important. For instance, the discount factor for a 12 months delay elicited under risk is equal to 0.82, corresponding to a yearly discount rate of 22%. The equivalent discount factor for a 12 months delay elicited from TEs under certainty is 0.68, corresponding to a yearly discount rate of 47%—more than double the discount rate estimated under risk. Beyond these differences, however, we also find the discount factors to be significantly correlated in all cases, indicating that they are capturing one and the same behavioral trait. These correlations are indeed strong, with the exception of the one month delay, where the correlation is only moderate to weak.



(a) CDF of utility parameters

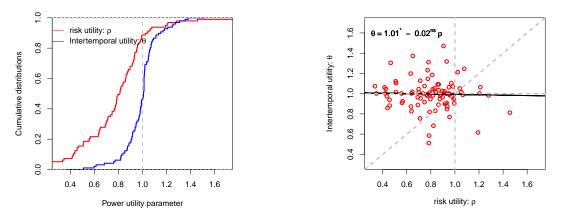
(b) Utilies under risk and over time

Figure 6: Utility parameters under risk and over time

## 3.3. Utility under risk and utility over time under GDEU

We now abandon the DEU assumption and move on to testing our generalized model. The first step will be to test for utility differences between choices over time and under risk. We assume CES utility functions for risk and time, i.e.  $u(x) = x^{\rho}$  and  $v(x) = x^{\theta}$  (Epstein and Zin, 1989; Miao and Zhong, 2015), and estimate  $\theta$  from Eq. (11) and  $\rho$  from Eq. (12). The equality of the power coefficients of the two utility functions under risk and time,  $\rho = \theta$ , is clearly rejected in our data (z = -2.953, p < 0.001; Wilcoxon signedrank test), with median values of  $\rho = 0.93$  and  $\theta = 1.01$ . Inter-temporal utility is thus different from risky utility, in violation of DEU and as predicted by GDEU. Indeed, we cannot reject the hypothesis that inter-temporal utility is linear (z = -0.170, p = 0.864; Wilcoxon signed-rank test). Risky utility, on the other hand, shows significant concavity (z = -5.228, p < 0.001; Wilcoxon signed-rank test).

Figure 6 compares the distribution of the utility parameters for risk,  $\rho$ , and the utility parameter for time,  $\theta$ . Panel 6a shows the cumulative distribution functions of the two utility parameters. For inter-temporal utility, we observe a large proportion of subjects having a parameter value close to 1. For risky utility, close to 60% of subjects have a parameter value below 0.9. Overall, there is thus a clear trend of more concave utility under risk than over time. Panel 6b shows a scatter plot of the two utility parameters  $\theta$  and  $\rho$ . The majority of the data points for the risky utility parameter  $\rho$  can be seen to be to the left of the dashed line at 1, indicating concave utility. For the intertemporal utility parameter



(a) CDF of utility parameters

(b) Scatter plot of utility parameters

Figure 7: Utility parameters under risk and over time with EU for risk

 $\theta$ , shown on the ordinates, there is no clear pattern, with about equally many points to either side of the dashed line indicating 1. The graph furthermore clearly shows that there is no correlation between inter-temporal and risky utility ( $r_s = 0.07, p = 0.466$ ; Spearman rank correlation). These results are in agreement with earlier findings (Andreoni and Sprenger, 2012a; Abdellaoui et al., 2013), and suggest that utility curavture under risk and intertemporal elasticity of substitution constitute independent behavioral characteristics.

We have so far considered our full GDEU model. There may, however, be some interest in considering the case of w(p) = p instead, i.e. the generalized model without probability weighting. Indeed, estimating risky utility while imposing linearity in probabilities generally results in different utility estimates from the full model (Bleichrodt et al., 2007; Booij et al., 2010). Figure 7 depicts the utility functions under the EU modeling assumption, with panel 7a showing the cumulative distribution function of the two utility powers, and panel 7b showing a scatter plot. The results are qualitatively similar to those seen above, except that utility under risk is even more clearly concave (z = -6.49, p < 0.001; Wilcoxon signed-rank test), with a median parameter value of  $\rho = 0.81$ . The scatter plot once again reveals no significant correlation between the two utility parameters ( $r_s = 0.051, p = 0.607$ ; Spearman correlation), thus contradicting the DEU assumption of intertemporal elasticity of substitution being the inverse of utility curvature under risk.

## 3.4. Correcting discounting for risk preferences

We have seen in the last two sections that risk counts—utility under risk is different from inter-temporal utility in our data, and discount functions elicited under risk and in riskless environments differ systematically. We will now investigate whether adjusting discounting for risk preferences, as postulated by our generalized model, helps to close the gap between risky and riskless discounting. Figure 8 shows again the comparison of discount factors obtained from the risky and the riskless tasks as shown in figure 5, and further adds the risky discount factors corrected for risk preferences (i.e., corrected using  $\phi$  and w according to Eq. (8)). The discount factors obtained under risk are generally lowered by the correction. This also means that the median corrected discount factors appear to fall much closer to the median discount factors obtained under certainty than the uncorrected discount factors obtained from risk equivalents.

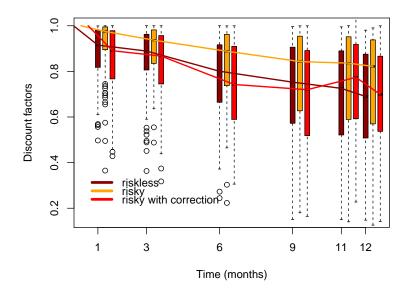


Figure 8: Discount functions under certainty, risk, and risk corrected for  $\phi$  and w

Table 4 further provides summary statistics of the discount factors and tests for differences between the corrected risky discount factors and the riskless discount factors. The median discount factors can now be seen to be very similar for corrected risky and riskless measures. Indeed, only one pairwise comparison, for  $\delta_{11}$ , of risky and riskless discounting is significant according to a Wilcoxon signed-rank test.<sup>10</sup> Using a Sign test, none of

<sup>&</sup>lt;sup>10</sup>The estimation of  $\delta_{11}$  may be seen as generally less stable than the estimation of the other discount

the pairwise comparisons result significant. This is furthermore before any adjustment for multiple testing are applied, with no test meeting the more stringent criterion of  $p \leq 0.008$ required by Bonferroni adjustments. In other words, we cannot reject the hypothesis that the discount factors are the same once we correct discount factors obtained from REs for risk preferences using both  $\phi$  and w. This stands in marked contrast to the finding for the uncorrected discount factors obtained from REs, which were found to be significantly larger than the riskless ones in all cases. The correlations between the discount factors remain generally strong and highly significant after the correction, and are of the same order of magnitude as observed before the correction.

	Risky corrected for $\phi$ and $w$		Riskless		Comparison			
	Median	IQR	Median	IQR	Wilcoxon test	Sign test	Spearman Correlation	
$\delta_1$	0.89	[0.77; 0.98]	0.91	[0.82; 0.98]	$-1.518^{ns}$	$49/55^{ns}$	0.376***	
$\delta_3$	0.87	[0.75; 0.96]	0.88	[0.80; 0.95]	$-1.852^{ns}$	$44'/60^{ns}$	$0.639^{***}$	
$\delta_6$	0.74	[0.59; 0.91]	0.79	[0.64; 0.91]	$-1.414^{ns}$	$47/57^{ns}$	$0.652^{***}$	
$\delta_9$	0.72	[0.52; 0.89]	0.74	[0.56; 0.88]	$-0.513^{ns}$	$52'/52^{ns}$	$0.612^{***}$	
$\delta_{11}$	0.77	[0.59 ; 0.92]	0.72	[0.57; 0.88]	$1.965^{ns}$	$58'/46^{ns}$	$0.618^{***}$	
$\delta_{12}$	0.70	0.54 ; 0.87	0.68	[0.56; 0.84]	$0.152^{ns}$	$55'/49^{ns}$	$0.597^{***}$	

Table 4: Statistics on discount parameters corrected by  $\phi$  and w

p-values in parentheses; all p-values reported are two-sided; \*: p < 0.05; \*\*: p < 0.01; \*\*\*: p < 0.001; ns: non significant

To complete the picture on the effect of correcting discount factors elicited under risk for risk preferences, we can take a look at their distributions. Figure 9 shows the cumulative distribution function of the six discount factors. Each panel compares the cumulative distribution function of the riskless discount factor to those of the uncorrected risky discount factor and the risky discount factor corrected for risk preferences. A common pattern emerges across the panels. Compared to riskless discounting obtained from TEs, the uncorrected discounting obtained from risky tradeoffs generally shows an accumulation of observations close to 1. Correcting risky discounting for risk preferences eliminates this accumulation close to 1, and pushes the distribution close to the one observed for riskless discounting.

An interesting question is whether we need our full model, or whether it is sufficient to estimate the model assuming that probabilities are treated linearly, i.e. w(p) = p. A boxplot comparing the risky discount factors corrected by  $\phi$  under the assumption that

factors, inasmuch as it is identified purely from a comparison with s = 11 and  $\ell = 12$ , which was inserted for the identification of hyperbolic behavior in comparison to the 1 month delay from s = 0. This means that this discount factor is defined only in contrast to the 12 months discount factor, and is thus not fully independent.

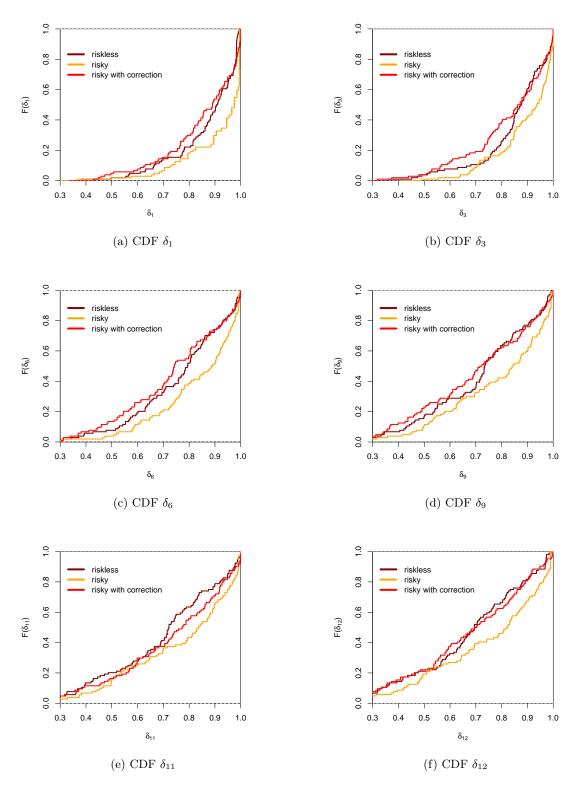


Figure 9: Cumulative distribution functions of discount factors, corrected and uncorrected

w(p) = p to the riskless discount factors and the uncorrected risky discount factors is shown in figure 10. Correcting the risky discount factors using  $\phi$  clearly lowers them, making discounting more severe and hence closing the gap with riskless discount factors. At the same time, this correction does not go far enough, with discount factors under risk still being systematically larger than riskless discount factors even after applying the correction.

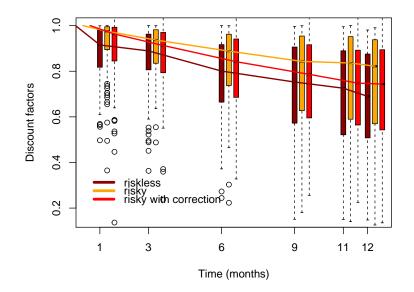


Figure 10: Discount functions under certainty, risk, and risk corrected for  $\phi$  under w(p) = p

To see if this intuition gathered from the boxplot is indeed correct, table 5 presents summary statistics on the discount factors and tests their equality. The median discount factors can be seen to remain universally larger under risk than they are under certainty. This difference remains significant for three discount factors according to the signed-rank test (and marginally significant for two more), and for five out of six according to the sign test. This difference furthermore remains significant for two comparisons after applying Bonferroni adjustments according to the Wilcoxon test, and for fully five comparisons when applying the sign test. The difference also remains significant economically. The 12 months discount factor elicited in the absence of risk is 0.68, corresponding to a yearly discount rate of 47%. The corresponding discount factor obtained from the risky measure after correcting for  $\phi$  is 0.74, corresponding to a yearly discount rate of 35%. Although the gap has narrowed somewhat relative to the case in which no correction was applied (where the yearly discount rate obtained from REs was 22%), there continues to be a sizable difference in estimated discount rates.

	Risky corrected for $\phi$		Riskless		Comparison				
	Median	IQR	Median	IQR	Wilcoxon paired test	sign test	Spearman Correlation		
$\delta_1 \ \delta_3 \ \delta_6 \ \delta_9 \ \delta_{11} \ \delta_{12}$	$\begin{array}{r} 0.96 \\ 0.91 \\ 0.84 \\ 0.79 \\ 0.75 \\ 0.74 \end{array}$	$\begin{bmatrix} 0.85 ; 0.99 \\ 0.80 ; 0.97 \\ 0.69 ; 0.94 \\ 0.60 ; 0.91 \\ 0.57 ; 0.89 \\ 0.54 : 0.89 \end{bmatrix}$	$\begin{array}{c} 0.91 \\ 0.88 \\ 0.80 \\ 0.74 \\ 0.72 \\ 0.68 \end{array}$		$\begin{array}{c} -1.899 \ (0.058) \\ -1.159 \ (0.246) \\ -2.979 \ (0.003) \\ -2.298 \ (0.022) \\ -1.837 \ (0.066) \\ -2.638 \ (0.008) \end{array}$	$\begin{array}{c} 65/39 \ (0.007) \\ 59/45 \ (0.101) \\ 68/36 \ (0.001) \\ 65/39 \ (0.007) \\ 68/36 \ (0.001) \\ 69/35 \ (0.001) \end{array}$	$egin{array}{l} 0.399\ (<\!0.001)\ 0.630\ (<\!0.001)\ 0.738\ (<\!0.001)\ 0.742\ (<\!0.001)\ 0.608\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.001)\ 0.656\ (<\!0.$		

Table 5: Statistics on discount parameters corrected by  $\phi$  assuming w(p) = p

p-values in parentheses; all p-values reported are two-sided

## 3.5. Patterns of discounting

We now discuss the shape of discount functions. In keeping with the analysis above, we pursue a semi-parametric approach whereby we parametrically estimate only the correction factors v,  $\phi$ , and w, while we base the analysis of the shape of discount functions purely on a nonparametric analysis of the discount factors. This is important inasmuch as imposing specific functional forms could distort discounting even after controlling for other potential distortions such as the one deriving from nonlinear utility. We will focus on riskless discounting and discounting under risk corrected as prescribed by the GDEU model, since we already have examined discounting under risk assuming DEU in section 3.1 in a fully non-parametric way.

We start by testing whether exponential discounting may be a good model. We test for hyperbolicity by comparing the discount factor estimated for a given delay t + k to the discount factor estimated for the delay t, taken to the power (t + k)/t. For instance, the discount factor for t + k = 6 months is compared to the discount factor for t = 3 months, taken to the power (t+k)/t = 2. If discounting is exponential, the two should be identical. If we observe hyperbolic behavior, on the other hand, we would expect that  $\delta_{06} > \delta_{03}^2$ , indicating that discounting for a 6 months period extrapolated from the discount factor for the first 3 months overestimates the severity of discounting.

Table 6 shows the discount patterns for both time equivalents and corrected risk equivalents, as well as tests for the relative frequency of different types of behavior. The pattern based on the discount factors derived from time equivalents is very clear. In all cases the discount factors based on shorter time delays are smaller than the actual discount factors measured over the whole delay, indicating hyperbolic discounting. This does not, how-

$\delta_{t+k}$		$\delta_{12}$		δ	9	6	$\delta_6$	
$\delta_t^{\frac{t+k}{t}}$	$\delta_1^{12}$	$\delta_3^4$	$\delta_6^2$	$\delta_1^9$	$\delta_3^3$	$\delta_1^6$	$\delta_3^2$	
$\delta_{t+k} > \delta_{t}^{\frac{t+k}{t}}$	89	69	75	89	78	91	61	
$\delta_{t+k} < \delta_t^{\frac{t+k}{t}}$	15	35	29	15	26	13	43	
signed-rank	z = 7.74	z = 4.50	z = 5.36	z = 7.76	z = 4.81	z = 7.44	z = 2.62	
test	p < 0.001	p < 0.001	p < 0.001	p < 0.001	p < 0.001	p < 0.001	p = 0.009	
		actors estima	ated from ris	k equivalents	(corrected for	$\phi \text{ and } w$		
$\delta_{t+k} > \delta^{\frac{t+k}{t}}$	85	77	77	86	70	85	60	
$\delta_{t+k} < \delta^{\frac{t+k}{t}}$	19	27	27	18	34	19	44	
signed-rank	z = 7.33	z = 5.81	z = 6.07	z = 7.17	z = 4.50	z = 6.99	z = 3.03	
test	p < 0.001	p < 0.001	p < 0.001	p < 0.001	p < 0.001	p < 0.001	p = 0.002	

discount factors estimated from time equivalents

ever, tell us whether the pattern we find is truly hyperbolic or quasi-hyperbolic, since all comparisons include the delay from t = 0. To address this issue, we can construct three months delays that are purified of the discounting of the initial period, by taking  $\delta_{t+k}/\delta_t$ , where k is equal to a 3 months delay. Comparing  $\delta_6/\delta_3$  to  $\delta_9/\delta_6$ , we find that discounting is still considerably stronger in the earlier measure (z = 4.26, p < 0.001). This provides some evidence for truly hyperbolic behavior. Comparing  $\delta_9/\delta_6$  to  $\delta_{12}/\delta_9$ , on the other hand, we find no significant difference (z = -1.05, p=0.295). While there is thus some evidence for strongly hyperbolic behavior, this evidence remains weak for later time delays and quasi-hyperbolicity seems to be the predominant pattern.

The discount factors obtained from risk equivalents paint a very similar picture to the one just seen for time equivalents. Just as seen for the latter, a clear majority of subjects displays larger discount factors for the longer delays than one would impute from the earlier delays. The similarity also carries over to the strength of hyperbolicity. Comparing  $\delta_6/\delta_3$  to  $\delta_9/\delta_6$ , we once again find that discounting is still stronger in the earlier measure (z = 2.18, p=0.029). And once again, this hyperbolic pattern disappears when comparing  $\delta_9/\delta_6$  to  $\delta_{12}/\delta_9$  (z = 0.08, p=0.93). We thus conclude that while there is clear evidence for some sort of hyperbolicity in the data, this seems mostly due to the earliest delays, likely indicating quasi-hyperbolicity. The evidence for strongly hyperbolic patterns is weaker, and furthermore seems to decline with longer time delays, disappearing for delays longer than 6 months.

## 4. Discussion and conclusion

Under DEU, discounting is supposed to be invariant with respect to the presence or absence of risk. However, the substantial empirical evidence showing the descriptive importance of non-linear probability weighting under risk (Tversky and Kahneman (1992); Wakker (2010)), and the more recent evidence uncovering differences between utility under risk and intertemporal substitution (Abdellaoui et al., 2013; Andreoni and Sprenger, 2012a), suggest that one could reasonably expect that discounting could differ depending on whether the latter was elicited from risky or risk-less intertemporal tradeoffs. The paper tests the hypothesis that correcting for non-linear probability weighting and distinguishing risky utility and intertemporal substitution within DEU allows us to observe the same discount function ragrdless of whether intertemporal tradeoffs were risky or riskless. The risk equivalent method was key to show how the aforementioned choice anomalies affect discounting under DEU, inasmuch as it naturally integrates the risk and time domains.

Specifically, the risk equivalent method allows us to bypass the utility confound in the measurement of discounting under DEU (Frederick et al., 2002), and to thus access the discount function directly through the elicitation of REs. Comparing the discount factors thus obtained to discount factors obtained from riskless intertemporal tradeoffs, we found systematic differences between the two. One of the reasons for this may be differences between risky and intertemporal utility. While we indeed found the two utility functions to be different, thus confirming previous results, we also showed that this difference could not fully account for the difference in discounting. Correcting the discount factors obtained under risk for both utility differences and probability weighting, however, made the differences between the two disappear. This indicates the importance of combining utility differences  $\phi$  and probability weighting w to account for the behavioral patterns in the data.<sup>11</sup> Indeed, neglecting the difference between the two utility functions and/or probability weighting in the elicitation of discounting could lead to a biased estimate of discounting Abdellaoui et al. (2010); Andersen et al. (2008).

Our approach to the elicitation of discounting is semi-parametric in the sense that no parametric form was imposed on the discount function, i.e. only utility and probability weighting functions were assigned parametric forms. Using REs under DEU, our nonparametric analysis at the individual level showed about one third of subjects to exhibit constant discounting. The modal pattern for delays from the present, however, consisted in subjects exhibiting decreasing impatience. This tendency towards decreasing impatience declined markedly when comparing delayed consequences, indicating a quasi-hyperbolic rather than fully-hyperbolic pattern in the data. A sizable minority of subjects also exhib-

<sup>&</sup>lt;sup>11</sup>This result is consistent with the recent experimental findings in Dean and Ortoleva (2016). The authors found a significant correlation between attitudes towards risk (as represented in our model by  $\phi$  and w) and discount rates.

ited the opposite pattern of increasing impatience (Attema et al., 2010). These findings were not due to the use of REs under DEU alone. Indeed, we replicated the same type of (quasi-) hyperbolic patterns both with our corrected risk equivalents and with time equivalents obtained in riskless environments.

Behavioural research suggests that– decisions involving tradeoffs under risk and over time exhibit remarkably similar violations of DU and EU by Prelec and Loewenstein (1991). This seems to derive from the common intuition that both risk and time create a similar psychological distance from the desired outcome (Dean and Ortoleva, 2016; Köszegi and Szeidl, 2013). The latter, however, cannot provide an account of the difference between risky and riskless discount functions obtained in the present paper. We exploit this intuition in our experimental task by having subjects trade off the two elements of psychological distance against each other, thus naturally integrating risk and time in our design.

While we are not the first to account for probability weighting in intertemporal choice (Epper and Fehr-Duda, 2014; Halevy, 2008; Miao and Zhong, 2015), our work is the first to account for *both* domain-dependent utility and probability weighting under risk, and to precisely measure their respective impact on the difference between discounting in risky and riskless intertemporal tradeoffs setups. In situations where risk is explicitly mixed with intertemporal tradeoffs, such as the ones we investigate, utility has been shown empirically to differ between risky and riskless intertemporal tradeoffs. This makes the modeling of differences in utility across domains imperative. Nevertheless, our results have also shown that allowing for such differences in utility alone is not sufficient to account for differences in discounting between risky and riskless environments. To achieve that aim, allowing for both differences in utility and nonlinear probability weighting appears to be essential. Once both differences in utility and nonlinear probability weighting are accounted for, however, we can conclude that the discount functions under risk and in riskless environments are the same-in other words, there is only one discount function.

## A. Econometric approach and parametric specifications

All the stimuli involve equivalences, either REs, CEs or TEs, measured by choice lists. We obtain a *predicted equivalent*,  $\hat{q}$ , for a given prospect  $\boldsymbol{p}$ . The predicted equivalent will be a function of the prospect  $\boldsymbol{p}$  and of the vector of all model parameters  $\boldsymbol{\omega}$ , indicated as  $\hat{q}(\boldsymbol{p}, \boldsymbol{\omega})$ . The predicted *time* equivalent,  $\hat{q}_t(\boldsymbol{p}, \boldsymbol{\omega})$ , takes the following form

$$\hat{q}_t(\boldsymbol{p}, \boldsymbol{\omega}) = v^{-1} \left[ \frac{\delta_\ell}{\delta_s} \left( v(x) - v(x') \right) + v(y) \right]$$
(11)

We implement intertemporal utility v as a power function with constant elasticity of substitution (*CES*), where  $v(x) = x^{\theta}$ . The CES function is the most popular function in the literature (Brown and Kim, 2014; Miao and Zhong, 2015), as well as being convenient to work with in the present setup.

The predicted *certainty* equivalent,  $\hat{q}_c(\boldsymbol{p}, \boldsymbol{\omega})$ , will take the following form

$$\hat{q}_c(\mathbf{p}, \boldsymbol{\omega}) = (\phi \circ v)^{-1} \left[ w(p) \left( \phi[v(x)] - \phi[v(y)] \right) + \phi[v(y)] \right]$$
(12)

We again need to specify parametric forms for our function. Following the approach taken for v, we will implement  $\phi$  as a power function, and designate the power parameter by  $\gamma$ . This means that  $\phi \circ v$  will also follow a power specification, with a parameter  $\rho = \gamma \theta$ . For the probability weighting function w we adopt the one-parameter function proposed by Prelec (1998):

$$w(p) = exp(-(-ln(p))^{\alpha})$$
(13)

In our aggregate data, we cannot reject the hypothesis that the one-parameter function performs as well as the two-parameter formulation ( $\chi^2(1) = 0.37, p = 0.544$ , likelihood ratio tests). At the individual level, the 2-parameter formulation outperforms the one-parameter formulation only for 29 subjects. Since the 2-parameter version generates outliers due to collinearity between the pessimism parameter and utility, we adopt the 1-parameter function also at the individual level.

Finally, the predicted *risk* equivalent,  $\hat{q}_r(\boldsymbol{p}, \boldsymbol{\omega})$ , will take the following form

$$\hat{q}_r(\boldsymbol{p}, \boldsymbol{\omega}) = w^{-1} \left[ \frac{\phi(\delta_\ell)}{\phi(\delta_s)} \right], \tag{14}$$

where the righthand side is now transformed by the inverse of the weighting function,  $w^{-1}$ , since the predicted outcome takes the form of a probability.

In our data, we have an observed equivalent, q, which may be different from the theoretically predicted equivalent,  $\hat{q}$ , due to errors in responses or model mis-specifications relative to the true underlying decision process. To account for this, we add an error term to the predicted equivalent, such that  $q = \hat{q} + \epsilon$ , where  $\epsilon$  is assumed to be a normally distributed error with mean 0 and variance  $\sigma^2$ ,  $\epsilon \sim \mathcal{N}(0, \sigma^2)$  (Hey and Orme, 1994). To allow for heteroscedasticity, we estimate one error term for each prospect type, with standard deviations  $\sigma_t$  for TEs,  $\sigma_c$  for CEs, and  $\sigma_r$  for REs. We furthermore make  $\sigma_c$  proportional to the outcome range in a choice list (Bruhin et al., 2010), which provides a significantly better fit to our data than the homoscedastic version (z = 35.75, p < 0.001, Vuong (1989) test)<sup>12</sup>. No such adjustment is needed for  $\sigma_r$ , since  $q_r$  is naturally contained in the probability interval. As to  $\sigma_t$ , a formulation allowing for heterscedasticity by the choice list range is rejected in favor of a homoscedastic formulation (z = -115, 54, p < 0.001, Vuong test), which we thus adopt.

We use maximum likelihood estimation to obtain our parameter values. Since our equivalents are measured in choice lists, we obtain interval information on each equivalent q. That is, we know that the true equivalent falls somewhere in the interval of outcomes between which a subject switched between prospects, but not exactly where in the interval it falls. To reflect this in our econometric approach, we define a lower bound  $q^-$  and an upper bound  $q^+$  for our equivalent q, such that  $q^- \leq q \leq q^+$ . The cumulative probability distribution function,  $\Pi$ , associated with a given choice will then be

$$\Pi(\mathbf{p}, \boldsymbol{\omega}) = P(q^{-} \le q \le q^{+}) = P(q^{-} - \hat{q} \le \epsilon \le q^{+} - \hat{q}) = \Phi(\frac{q^{+} - \hat{q}}{\sigma}) - \Phi(\frac{q^{-} - \hat{q}}{\sigma}) \quad (15)$$

where P indicates the probability of q falling into a given interval, and  $\Phi$  designates the cumulative normal distribution function. The formulation in terms of upper and lower bounds on the probability function has the advantage that it explicitly deals with corner

<sup>&</sup>lt;sup>12</sup>A nonparametric Clark test results in the same qualitative conclusion; unless there are differences between the two tests, this will not be further mentioned.

solutions, i.e. subjects who consistently chose one and the same option in a choice list. The resulting log likelihood, LL, obtains by taking logs and summing over prospects

$$LL(\boldsymbol{p},\boldsymbol{\omega}) = \sum_{i=1}^{N} \log \left[ \Pi_i(\boldsymbol{p},\boldsymbol{\omega}) \right]$$
(16)

where  $\Pi_i$  represents the cumulative probability function for prospect *i* from Eq. (15). We estimate this likelihood at the individual level using the BFGS algorithm.

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ONLINE APPENDIX (To be published online only)

## A. Correcting discount factors using previous estimates of risk preference parameters

We now show how to correct our discount factors for typical risk preference values from the literature. To this extent, we use the data obtained in Germany by L'Haridon and Vieider (2016). The data have the advantage that they were obtained using a similar methodoloy to the one used in this paper, as well as in the same country and using similar stake sizes. We re-estimate their model using the same functional forms as in this paper, and obtain  $\rho = 0.928$  and  $\alpha = 0.677$ . Table 7 reports the corrected discount factors, and compares them to the discount factors obtained from TEs in the absence of risk. The values can be seen to be very similar. Indeed, only one discount factor remains significantly different after a correction according to the Wilcoxen test. Two remain significant at conventional values according to the sign test. While none of the sign tests survive a Bonferroni adjustment, the one difference recorded by the significant Wilcoxon test remains significant even after the Bonferroni adjustment.

Risky co	crected for $\phi$ and $w$	Riskless		Comparison	
Median	IQR	Median	IQR	Wilcoxon paired test	Sign test
0.915	0.818; 0.982	0.914	0.784; 0.971	-1.688 (0.091)	$41/\ 63\ (0.019)$
0.889	[0.807; 0.96]	0.844	[0.713; 0.93]	-2.865(0.004)	$41/\ 63\ (0.019)$
0.801	[0.665; 0.916]	0.775	[0.62; 0.888]	-1.727(0.084)	$44/\ 60\ (0.07)$
0.753	[0.574; 0.905]	0.722	[ 0.526; 0.871 ]	-1.182(0.237)	$46/\ 58\ (0.14)$
0.726	[0.523; 0.889]	0.714	[0.501; 0.866]	-0.495(0.621)	$50/\ 54\ (0.384)$
0.69	[0.509; 0.875]	0.699	[0.483; 0.843]	-0.384(0.701)	48/56~(0.246)

p-values in parentheses; all p-values reported are two-sided.

Table 7: Correction using risk preference parameters from L'Haridon and Vieider (2016)

## **B.** Instructions to the participants

## Instructions

Thank you for participating in this experiment in decision making! You will obtain a fixed payment of 5 Euros for your participation in this experiment. Those 5 Euros are yours to keep independently of the outcomes in the experiment. In addition, you will be compensated with whatever you earn during the experiment according to the procedures described in these instructions.

Detailed instructions are given in the video you just watched. This note summarizes the instructions. You may consult these instructions at any time during the experiment. An experimenter will be seated next to you during the experiment, ready to assist you in case you should have any questions or doubts at any point during the experiment. We are only interested in your preferences: there are no right or wrong answers!

You will be asked to make a number of different choices. Please consider each decision carefully. Take a careful look at the outcomes, the probabilities, and the payment dates associated to them before taking a decision. Remember that your final payoffs from this experiment will depend on the decisions you take (and of course, on chance). There is no need to rush: the experiment has been designed to be easily completed in an hour, and it will not be possible to leave the experiment before the end of the session.

Please remain seated when you are finished with the tasks, the experimenter will proceed to the payout with you. After this, you will be paid your participation fee, plus any additional amount you may have won.

Do not talk during the experiment to anyone other than the experimenter, or you will be immediately excluded.

Good luck!

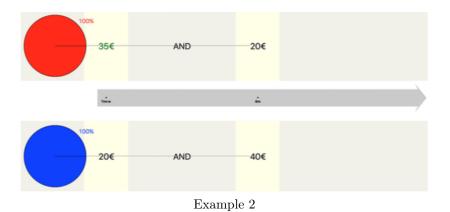
#### **Decision Task**

In this experiment, you are asked to indicate your preferences between situations involving risk and/or time delays. A typical choice is depicted below:





The blue option offers 50 Euros for sure, payed in 9 months from now. The red option offers a probability of 65% to obtain these 50 Euros tomorrow, and a complementary probability of 35% to receive nothing.



In the example above, situations involve one payment only. In other decision tasks, however, situations may involve multiple payments. An example of this is depicted here below:

The red option and the blue option offer certain streams of two payments. In both cases, a first payment would be received tomorrow and an additional payment would be received in 6 months from now. The amount of the payments vary across options.

A third and last type of questions involves risk only, as in the following example.



Example 3

In Example 3, the outcomes of the two lotteries are payed tomorrow. The outcome of the red lottery is sure: 25 Euros. The blue lottery offers 50% chance to receive 40 Euros, and 50% chances to receive 10 Euros.

These three examples illustrate all the dimensions involved in the decision tasks, and that you should *carefully check* for each choice: different *outcomes* for each of two situations; the *probability* of winning the higher amount within each option, and the *time* at which the indicated amounts are to be paid out, which may or may not differ between the two options.

#### The choice lists

The choices are grouped within lists. Within a list, the blue option is fixed, while the red option changes. When the red option option is la lottery, as in Example 1, its probability varies within choice list; when the red option offers a sure outcome, this outcome varies within choice list.

Take a look again at Example 1. If the red option offers 50 Euros with a probability of 100%, you will want to choose it because the 50 Euros are received for sure as in the blue option, but sooner. If the red option offers 0% chances to get the 50 Euros, you will want to choose the blue option because receiving something, although in the future, is better than nothing. Taken together, this indicates that you will want to switch from the red option to the blue option at some

point as the probability of the red option changes. Where you switch depends entirely on your preferences—there are no right or wrong answers.

Take again a look at the Example 3 above. If the red option offers 10 Euros, you will want to choose the blue option, because 10 Euros is the worst outcome of the blue option. If the red option offers 40 Euros, you will want to choose it because 40 Euros is the best outcome of the blue option. Taken together, this indicates that you will want to switch from the blue option to the red option at some point as the outcome of the red option increases. Where you switch depends entirely on your preferences— there are no right or wrong answers.

In order to help you save time in completing the choice list, the software will initially present you with some isolated choices from the list and then fill in the list according to your preferences. Looking again at Example 3, a preference for the red over the blue option for a value of 25 Euros means that you would also prefer the red option for larger amounts. In that case, the software would select the red option for an outcome of 25, and would also complete the choices, selecting the red option for outcomes ranging from 26 Euros to 40 Euros. The same process applies to probabilities when the red option is a lottery.

It is important to understand that this is merely a procedure devised to speed up the choice process. Once the complete list has been filled in by the procedure, you will be asked to double-check your choices by directly looking the list including all different amounts offered by the red option. At this point, you should check whether the indicated choice does indeed correspond to your preferences for each choice. If this is not the case, you can still change any decisions that do not correspond to your preferences by moving the ruler. Once you are satisfied with your choices, you can confirm this and move on to the next choice list. Please notice that there will be no possibility to come back to a choice once you have confirmed the final choice list. It is important that you are satisfied with all the choices from the list, because all the choices are eligible from real payment.

#### Payment mechanism

You will receive a flat payment of 5 Euros for your participation in the experiment. This payment is independent of your choices, and will be given to you as soon as you finish the experiment and respond to all choices. In addition, you may play out one of your choices in the task for real. It is thus important that you pay close attention to all the aspects in all decision tasks, as these may determine the level and timing of your additional compensation.

At the end of the session, one of the choices you make will be selected and you will be paid according to the choice you made.

Here is the procedure for the selection of one choice for real payment. First, one of the choice lists in the experiment will be drawn at random by picking a numbered piece of paper from a bag. For the selected list, one of the choices will subsequently be randomly selected. For the selected choice, the option chosen by the subject during the experiment will then be played for real, again by drawing a numbered piece of paper from a bag. You may check the contents of the bags at any time during the procedure should you desire to do so. The decision will be played out immediately after the experiment, no matter of the payout data of the selected option.

Notice that only one choice will be played for real and that the selected lottery will be played only once. Since every single choice of choice lists has the same probability of being selected to be played for real, you should respond to every single choice as if it were the one you will ultimately play. In other words, there is no way in which you can increase your winnings or spread your risk by answering strategically. The optimal strategy is to fill out each single choice as if it were the one that will be selected for real play.

#### Procedure for delayed payout

Everybody will obtain the participation payment of 5 Euros immediately after the experiment. In case you should win any additional money, this amount will be paid to you via bank transfer. The bank transfer will be initiated on the day indicated on the decision screens. This means that the amount will be on your bank account 3 days after the payment is initiated. This means that any outcomes that obtain tomorrow will in fact be on your account in 3 days. A payout that obtains in 6 months will be on your account 6 months + 3 days from today.

To make the bank transfer possible, we will ask for your bank information if you are selected to play one of your choices for real. This information will be given to the administration of the Wissenschaftszentrum Berlin fÃŒr Sozialforschung (WZB), so that the transfer can be effectuated on the date indicated. This information will in no way be linked to your decisions in the experiment, but only to the final amount you won. Also, it will be destroyed once the transaction has taken place.

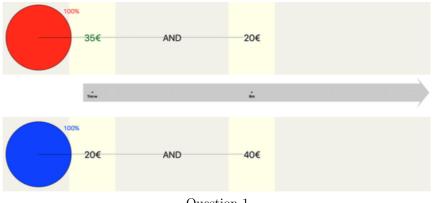
The Bank transfer will be executed by the WZB through Commerz Bank. Once the outcome has been determined, you will also be given an official certificate, stating that you are entitled to a given payout on a given date. This certificate constitutes a guarantee from the WZB that you will obtain that payout on the indicated date. The certificate also contains the contact details of the person responsible for the transfers at the WZB, whom you can contact in case you have any doubts about the procedures, or in case you banking details should change before the payment date.

#### **Final remarks**

Please take the time to read through the instructions again on your own. If you have any doubt, ask and the experimenter will help you. Once you understand how the choices work, you will have the opportunity to familiarize yourself with the software with a few practice questions. Only then will the real experiment start.

#### **Comprehension Questions**

Following is a series of 5 comprehension questions. For each question, select the only right answer: a, b or c.



Question 1

The blue option offers:

• (a) 20 Euros tomorrow and 40 Euros in 6 months

- (b) 50% chances to receive 20 Euros tomorrow and 50% chances to receive 40 Euros in 6 months
- (c) 35 Euros tomorrow and 20 Euros in 6 months;

<b>65</b> X	50€ 0€				
I	tere and a			4	
100%				50€	



The blue option offers :

- (a) 50€ for sure, payed by bank transfer initiated in 9 months
- (b) 50€ for sure, payed in cash at the end of the experiment
- (c) 50 experimental tokens, converted into cents for real payment in 9 months



Question 3

The red option offers:

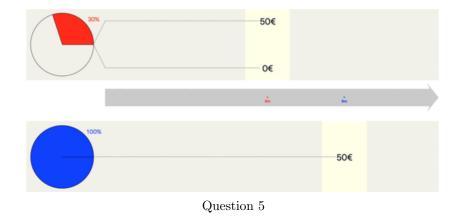
- (a) 65% chances to get nothing and 35% chances to get 50 Euros
- (b) 65% chances to get 50 Euros and 35% chances to get nothing
- (c) A higher probability to get 50 Euros than the blue option

If the blue is played for real, you will receive:



Question 4

- (a) nothing because payments are hypothetical
- (b) 50€ on your bank account in December 2016
- (c) 50€in cash in December 2016



If the screen above is selected for payment, and you chose the red option, when will you know the outcome of the lottery?

- (a) in 6 months, depending on the amount that appears on your account
- (b) tomorrow
- (c) at the end of the experimental session