Reply to: A Comment on "Decisions under Uncertainty as Bayesian Inference on Choice Options" by Kemel and Richard, August 2026 version

Ferdinand M. Vieider, 14 October 2026

Preliminary statement

I begin once more by briefly documenting the editorial process, to the extent that it is known to me:

- As acknowledged by the authors in their response to the previous round of reviews, they were not provided with my report on their earlier submission, although they subsequently discovered it on my personal website.
- In August 2026, Christoph Loch, Editor-in-Chief of *Management Science*, contacted me and invited me to review a new version of the comment.
- I submitted a detailed report. I have since learned that the comment has received a revise-and-resubmit decision, about which I was not informed. The editorial team handling the comment, led by George Wu, has provided me with no information about the subsequent proceedings.
- My explicit question to Dr. Loch as to whether I would be afforded an opportunity to respond should the comment be published has also once again remained unanswered.

Given the continued lack of transparency surrounding the editorial process, and the absence of any assurance that I will be permitted to respond to a published comment on my work, I consider it appropriate to make my assessment publicly available.

Summary and Assessment

The authors present a comment on the Bayesian Inference Model (*BIM*) of Vieider (2024). Their first result establishes that the primitive parameters of the BIM are not jointly identified from binary choices. From this they infer that the model's parameters cannot be meaningfully interpreted

in behavioural or cognitive terms. Their second result establishes an observational equivalence between the BIM and a stochastic version of Yaari's (1987) dual-EU model. From this they conclude that the BIM cannot account for diminishing sensitivity to outcomes and does not provide cognitive foundations for prospect theory.

The first point is simply odd. The original paper does not claim to identify all primitive parameters from choice data, nor does its theoretical contribution depend on doing so. The second has no foundation in either logic or epistemology: econometric identification is neither necessary nor sufficient to confer substantive meaning upon model parameters. The BIM is a generative cognitive model, whose parameters describe the mechanisms underlying choice behaviour and generate testable causal predictions. The third point—the purported equivalence to Yaari's dual-expected utility model—reveals the source of the misunderstanding. The authors approach the BIM as though it were a purely descriptive model, and consequently appear unable to distinguish the econometric recoverability of parameters from their substantive meaning. To someone with a hammer, everything looks like a nail. Finally, their main constructive "contribution"—an observationally equivalent reduced-form representation of the BIM—does not constitute an equivalent theory of how behaviour responds to changes in the decision environment. As such, it is largely beside the point and provides no novel theoretical insight.

Remarkably, the authors' revision does not engage with the central conceptual objections raised in my previous report, nor with the additional references I subsequently provided, which directly address the generative logic of the BIM and the causal role of its cognitive primitives. Rather than speculate about their motives or pursue the technical details of their additional analyses, I concentrate on three main issues:

1. **Econometric identification and its practical significance.** The authors establish formal non-identification of some model primitives, but substantially overstate its practical consequences. Their own simulations show remarkably strong parameter recovery under the hierarchical specification used in the original paper. More fundamentally, identification from observational choice data is neither necessary for substantive parameter meaning nor sufficient to establish the validity of a cognitive model.

2. **The cognitive meaning of model parameters.** BIM parameters describe components of a generative inference process and imply specific behavioural comparative statics. Experimental manipulations of coding precision and prior expectations provide direct tests of these

predictions, independently of whether the underlying primitives can be separately estimated from observational choice data.

3. **Observational equivalence versus theoretical equivalence.** The authors mistake a reduced-form representation of choices under fixed model parameters for an equivalent theory of behaviour. Their conclusions regarding Yaari's dual-EU model and the absence of diminishing sensitivity to outcomes ignore the central role of changes in coding precision and prior expectations in BIM.

I discuss these issues in turn, concentrating on the conceptual and logical problems in the authors' arguments rather than undertaking a detailed examination of their additional econometric exercises, which largely amount to demonstrating that parameters become easier to identify once the underlying cognitive mechanisms have been stripped from the model.

Cognitive meaning does not depend on econometric identification. The central conceptual error of the comment is to approach the BIM as though it were a purely descriptive model. In descriptive models, parameters are introduced to summarize observed behavioural patterns, and their interpretation is consequently tied to what can be recovered from choice data. The BIM, by contrast, is a generative cognitive model: its parameters represent primitives of a hypothesized information-processing mechanism, from which choice behaviour is derived. Their meaning comes from their role in that mechanism, not from their econometric recoverability. The authors appear unable to conceive of this distinction, and consequently mistake a limitation on parameter estimation for a limitation on the theoretical content of the model. BIM was never conceived primarily as an econometric identification exercise, and Vieider (2024) makes no claim that all its cognitive primitives can be separately identified from observational choice data. Nor is such identification necessary for the model's theoretical contribution.

The distinction is fundamental. Econometric identification is neither necessary nor sufficient for parameters to have substantive meaning. Suppose, for the sake of argument, that none of the BIM parameters were identifiable from observational choice data. This would not invalidate their cognitive interpretation, nor the behavioural comparative statics derived from them. A model can make precise, testable predictions about the causal effects of changes in its underlying primitives even when those primitives cannot be separately recovered from an observational likelihood. Indeed, this is precisely the logic of the experimental manipulations discussed below. The fact that the authors' own Online Appendix reports remarkably strong parameter

recovery despite formal non-identification makes their sweeping dismissal of the model's empirical usefulness all the more peculiar.

Conversely, identification does not establish substantive meaning. A fully identified descriptive model may summarize choice patterns accurately conditional on its assumptions, without however offering any explanation of the cognitive processes generating them. Consider a model of expected-value maximization with an additive error term: identifying its parameters does not establish that people actually maximize expected value, nor that the residual errors correspond to any particular cognitive mechanism. Econometric identification establishes what can be learned about a model's parameters from a given distribution of observations. It says nothing, by itself, about whether the model provides a valid explanation of behaviour.

The authors' failure to distinguish these questions runs throughout the comment. Their insistence on econometric identification as a prerequisite for behavioural interpretation makes sense only if all models are treated as descriptive representations of choice. The BIM was developed precisely to move beyond such representations and investigate the cognitive mechanisms generating the observed patterns.

Cognitive meaning of BIM parameters. I illustrate this with recent work that I pointed out to the authors, but with which the comment does not engage. Oprea and Vieider (2026) develop a more general BIM with separate coding noise for outcomes and probabilities, and experimentally manipulate each source of noise. The model makes sharp predictions about the resulting changes in behaviour. Increasing probability noise reduces probability sensitivity and increases relative risk aversion, as predicted. Increasing outcome noise instead increases probability sensitivity—through γ/α, as α falls—while again increasing relative risk aversion. Thus, both probability sensitivity and risk aversion respond systematically to experimentally manipulated representational noise, exactly as implied by the cognitive primitives of the BIM. Importantly, these effects are established non-parametrically, without requiring separate econometric identification of the underlying primitives.

Of course, manipulating coding noise is not the only way to test the model's primitives. Bouchouicha et al. (2025) directly manipulate prior means and derive clear, nonparametric implications for choice.[1] Other primitives have equally specific roles. Prior variance, for instance, contributes to choice

[1]The model in Bouchouicha et al. (2025) is formally that of Khaw et al. (2021); for the case studied there, its predictions coincide with those of BIM.

stochasticity; Vieider (2025), which I also shared with the authors, derives this formally.

PT vs dual-EU foundations. The authors note that Equation 13 in Vieider (2024) takes the form of the familiar linear-in-log-odds (LLO) probability-weighting function, and conclude from this that BIM provides foundations for dual-EU rather than prospect theory. This conclusion is wrong for several reasons. I focus on the main ones below, abstracting from the fact that the BIM also explains gain–loss risk taking in ways that dual-EU does not; see Bouchouicha et al. (2025).

The authors' characterization of the BIM microfounding dual-EU raises major conceptual problems. In their Yaari representation, g and d are reduced-form preference parameters. In BIM they are not primitives at all, but endogenous functions of cognitive and environmental primitives. In particular,

$$g = \gamma/\alpha, \qquad d = \delta^{1/\alpha} = \xi^{(1-\gamma)/\alpha}\zeta^{(\alpha-1)/\alpha}.$$

Variation in d may therefore arise from variation in γ, driven by noise in probability coding; from variation in α, driven by noise in outcome coding; or from variation in the environmental priors ξ and ζ. Treating (g, d) as primitive tastes therefore strips away precisely the mechanism BIM is designed to explain.

This distinction is not merely semantic. Precisely because g and d are functions of cognitive and environmental primitives, they need not be stable across choice environments. Oprea and Vieider (2026) illustrate this directly. One of their treatments increases outcome-coding noise by scaling up the numerical units in which otherwise identical values are expressed (see Frydman and Jin, 2022; Garagnani and Vieider, 2025, for the underlying logic and evidence). This manipulation increases relative risk aversion, exactly as predicted by the BIM.[2] Within the model, greater outcome-coding noise lowers α, which in turn increases the influence of prior expectations on choice. Thus, α is not a "utility-curvature" parameter—there is no utility function in the BIM. It is a cognitive parameter governing outcome discriminability which, through Bayesian updating, modulates the impact of prior expectations on risk taking. Oprea and Vieider (2026) derive these comparative statics formally and document them nonparametrically in the data.

[2]The same manipulation also *increases* estimated probability sensitivity, another comparative-static prediction of BIM: lowering α raises γ/α even when γ itself is unchanged.

The last illustration is particularly important. A fixed-parameter dual-EU model cannot accommodate systematic stake-dependent changes in relative risk attitudes. Such patterns are indeed a central reason why the dual-EU model is violated by data; see Scholten and Read (2014) and Bouchouicha and Vieider (2017) for careful discussions disentangling Markowitz's (1952) EU interpretation from the dual interpretation. BIM, by contrast, predicts such patterns naturally through changes in outcome-coding noise (see also Khaw et al., 2023 for a model specifically geared towards certainty equivalents).

The claim that the BIM "only" microfounds dual-EU therefore mistakes a reduced-form representation for the mechanism generating it. It ignores what the model parameters are, where they come from, and—most importantly—how they change with the cognitive environment. This distinction speaks to the heart of the noisy-cognition approach. Rather than treating the parameters of descriptive choice models as fixed preferences, the BIM seeks to explain them as outcomes of an underlying cognitive process. It thereby generates predictions about how observed choice patterns change with the informational and representational environment—precisely the predictions that a descriptive representation, however well identified, cannot provide on its own.

Conclusion and constructive remarks. Econometric identification of the primitive cognitive parameters is not required for the theoretical contribution of Vieider (2024), and its purported absence does not in any way whatsoever affect the behavioural and cognitive interpretation of BIM's parameters. Identifying those primitives could, however, be useful for empirical applications seeking to distinguish the underlying cognitive mechanisms. The present comment has nothing to contribute in this respect. Its proposed "solution" produces an identifiable reduced-form representation precisely by discarding the decomposition into the cognitive primitives whose identification is supposedly at issue. It therefore does not solve the interesting identification problem; it removes it.

A considerably more interesting question is what additional observables or experimental variation would allow the primitive cognitive mechanisms to be separately identified. This could involve targeted experimental manipulations, as in Oprea and Vieider (2026) or Bouchouicha et al. (2025), but also supplementary process measures such as reaction times or other forms of process tracing. Addressing this question could turn the authors' econometric observation into a substantive contribution. Instead, the comment draws sweeping conclusions about cognitive meaning that simply do not follow from

its identification result.

The authors' fundamental misunderstanding of the nature of the BIM points to a much broader issue. The recent turn towards cognitive modelling in behavioural economics is beginning to show how predictive content can be restored to behavioural models by explaining the processes underlying observed choice patterns. It is no coincidence that this development builds on precisely the psychophysical insights that inspired Kahneman and Tversky (1979) to develop prospect theory in the first place. In this sense, the cognitive approach can be understood as a Lucas critique of descriptive behavioural economics: preference parameters estimated in one decision environment need not remain stable when that environment changes. Understanding how behaviour changes therefore requires moving beyond descriptive functions towards the cognitive processes generating them. Models grounded in psychophysics and perceptual neuroscience can impose restrictions on how those processes respond to changes in the decision environment, and thereby generate predictions that constitute systematic violations of descriptive models with fixed preference parameters. The BIM is necessarily a stylized and incomplete attempt at such an account, focusing on particular aspects of cognitive processing. But it represents a step towards explaining behavioural regularities rather than merely describing them. Reducing it to an observationally equivalent descriptive model misses precisely this contribution.

References

Bouchouicha, Ranoua, and Ferdinand M. Vieider (2017) 'Accommodating stake effects under prospect theory.' *Journal of Risk and Uncertainty* 55(1), 1–28

Bouchouicha, Ranoua, Yuchi Li, and Ferdinand M. Vieider (2025) 'Loss-sensitivity versus loss-aversion.' Technical Report, Ghent University Discussion Papers

Frydman, Cary, and Lawrence J Jin (2022) 'Efficient coding and risky choice.' *Quarterly Journal of Economics* 136, 161–213

Garagnani, Michele, and Ferdinand M. Vieider (2025) 'Economic consequences of numerical adaptation.' *Psychological Science* 36(6), 407–420

Kahneman, Daniel, and Amos Tversky (1979) 'Prospect Theory: An Analysis of Decision under Risk.' *Econometrica* 47(2), 263 – 291

Khaw, Mel Win, Ziang Li, and Michael Woodford (2021) 'Cognitive imprecision and small-stakes risk aversion.' *The Review of Economic Studies* 88(4), 1979–2013

_ (2023) 'Cognitive imprecision and stake-dependent risk attitudes.' Technical Report, NBER Working Paper 30417

Markowitz, Harry (1952) 'The Utility of Wealth.' *Journal of Political Economy* 60(2), 151–158

Oprea, Ryan, and Ferdinand M. Vieider (2026) 'Information processing and the anomalies of risk and time.' Technical Report, Working Paper

Scholten, Marc, and Daniel Read (2014) 'Prospect theory and the "forgotten" fourfold pattern of risk preferences.' *Journal of Risk and Uncertainty* 48(1), 67–83

Vieider, Ferdinand M. (2024) 'Decisions under uncertainty as bayesian inference on choice options.' *Management Science* 70(12), 9014–9030

Vieider, Ferdinand M (2025) 'The Structure of Stochastic Choice.' *Working Paper*

Yaari, Menahem E. (1987) 'The Dual Theory of Choice under Risk.' *Econometrica* 55(1), 95–115