Statistical Foundations of Ambiguity Attitudes[*]

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Abstract

Daniel Ellsberg famously showed that people exhibit preferences for strength of information. Here, I build Bayesian statistical foundations for such preferences. Modelling perceived likelihoods as probability *distributions* allows the informational content of an uncertain event to impact behaviour. Modelling the decision process as a Bayesian inference problem about the true probability builds bridges to models of noisy cognition. This simple modelling framework reconciles approaches based on sets of priors with empirical evidence organized under two-stage models of ambiguity. It further provides a natural framework for learning and probabilistic updating, which I discuss.

> *As the relevant evidence at our disposal increases [...] we have a more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of an argument.*
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> *John Maynard Keynes (1921), Ch. 6*

1 Introduction

Take an urn containing 100 balls that are either red or green. Daniel Ellsberg (1961) described a choice regularity whereby most people prefer betting on an urn known to have 50 red and 50 green balls over betting on an urn in which the colour composition is unknown. This observation holds regardless of the winning colour—a phenomenon known as the Ellsberg *paradox*. This choice pattern challenges subjective expected utility theory (*EU*; Savage, 1954), since under EU it

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implies that probabilities of complementary events sum to less than one in the *ambiguous* urn.

Here, I present a model of decision-making under ambiguity based on the tool of Bayesian *statistics*. Other than in EU, probabilities are represented by probability *distributions*, encoding both the "best guess" at the underlying probability, and the confidence a decision maker (*DM*) has in this best guess. A DM reacts to the lack of confidence in a probability assessment by inferring the true underlying probability by a convex combination of the best guess with her prior expectation. The problem thus takes the form of a Bayesian inference process about the true outcome-generating probability. This results in a highly tractable framework that characterizes ambiguity attitudes as endogenously arising from Bayesian statistical processing of vague information. Ambiguity attitudes thereby emerge purely as a product of vagueness in probability perception and Bayesian updating. The model further presents a natural foundation for learning and probabilistic updating to impact behaviour under uncertainty.

A generative model of decision-making under ambiguity. The model I present can best be thought of as *generative*: using the principled tool of Bayesian statistics, it generates predictions about choice behaviour starting purely from perceptions of ambiguity. The key intuition is one whereby subjects can hold belief distributions over ambiguous events. This provides an empirically tractable formalization of the idea that DMs base their decisions on beliefs that attribute strictly positive weights to a range of different probabilities, as proposed in multiple prior models (Gilboa and Schmeidler, 1989; Ghirardato, Maccheroni and Marinacci, 2004; Klibanoff, Marinacci and Mukerji, 2005).

The model yields behavioural predictions that emerge endogenously from the Bayesian combination of uncertain beliefs with prior expectations. This mechanism naturally predicts a number of empirical patterns that have been modelled in source theory (Baillon, Bleichrodt, Li and Wakker, 2025) and documented in the empirical literature: aversion to ambiguous probabilities in Ellsberg urns (Ellsberg, 1961), likelihood-dependence of ambiguity attitudes (Abdellaoui, Baillon, Placido and Wakker, 2011; Dimmock, Kouwenberg and Wakker, 2015), and reversals in ambiguity attitudes when moving from gains to losses Baillon and Bleichrodt (2015). Augmented by learning, the model predicts 'competence' to impact choice (Heath and Tversky, 1991).

Distinguishing features. The model I present nevertheless has a number of distinguishing characteristics that set it apart from the previous literature modelling ambiguity attitudes. Given its foundation in Bayesian statistics, behaviour is predicted to be inherently stochastic. Nevertheless, conditional on the uncertain probability perceptions, the Bayesian inference process devised to deal with with this uncertainty is *optimal*. The Bayesian setting furthermore provides a natural foundation for the integration of learning into the model, which is predicted to affect behaviour beyond the updating of average probabilities.

Perhaps the biggest departure from pre-existing models of ambiguity attitudes consists in the benchmark model of decision-making under risk. This departure arises from the observation that known probabilities are not a distinct category under the formalism of the model, but rather fall on a continuum of informational frictions in the coding of probabilities. The benchmark case thus obtains from models of "noisy cognition", which represent behaviour under risk as resulting from imprecise probability assessments (Zhang, Ren and Maloney, 2020; Khaw, Li and Woodford, 2023; Vieider, 2024). This deviates from the EU benchmark case in multiple prior models. It further provides cognitive microfoundations for probability weighting functions emerging endogenously from probability representations—a point to which I will return in the discussion.

2 The Psychophysics of Ambiguity

In what follows, I model binary choices between Ellsberg urns offering a prize x with an objective probability p or else 0. While I focus on such simple choices throughout to develop intuitions, the model generalizes to richer settings with multiple events attached to a distribution of outcomes.[1]

2.1 The model

In developing the model below, I will focus on the characterization of probabilities. This assumes implicitly that the common outcome or prize in Ellsberg-like choice problems is edited out of the choice problem. This is not a necessary feature of the model: if a DM has primitive *preferences* over outcomesor probabilities, the model can accommodate such additional motives. The point is rather to show that—in order to obtain the key insights accounting for ambiguity attitudes—the model does not need to assume the existence of such preferences: all the model parameters arise endogenously from probability perceptions and optimal ways to deal with them.

Modelling assumptions. Take a DM choosing between an urn known to contain 50 red and 50 green balls, and an ambiguous urn in which the exact proportion of the two colours is unknown. Assume the DM encodes her beliefs by two Beta distributions $p_a(x) = \mathcal{B}e(\alpha_a, \beta_a)$ and $p_r(x) = \mathcal{B}e(\alpha_r, \beta_r)$, where the subscripts a and r stand for ambiguity and risk, respectively. Further assume that the mean beliefs are identical for the two processes following exchangeability in winning colours (Raiffa, 1961), and that they correctly reflect the true probability p, so that $\widehat{p}_a = \widehat{p}_r \equiv p$, where $\widehat{p}_a \triangleq \mathbb{E}[\, p_a(x)\,] = \frac{\alpha_a}{\alpha_a+\beta_a}$ and $\widehat{p}_r \triangleq \mathbb{E}[\, p_r(x)\,] = \frac{\alpha_r}{\alpha_r+\beta_r}$. Additivity in mean beliefs is assumed without loss of generality: while this assumption is not necessary for the results below, it will allow us to zoom in on the key mechanism underlying the predictions we derive, and is thus mostly maintained for parsimony.[2]

A key mechanism driving choice in the model is the *confidence* a DM has in her

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[1] Given its foundation in perceptual psychophysics, highly complex situations would likely require a first stage during which the state space is simplified before the modelling insights here can be applied. See e.g Netzer, Robson, Steiner and Kocourek (2024) for a model of behaviour arising from different degrees of simplification of the state space.

[2] A slightly different interpretation could be that DMs observe signals indicating the true probability, which are constituted by "limited samples". The parameters of the Beta distributions can then be interpreted as counts of the successes and failures in these limited samples. Such an interpretation is consistent with the discussion of the relevance of the quantity of data by Eichberger and Guerdjikova (2013). In the case described here, however, such 'data' would be constituted purely by mental resources dedicated to coding the probability.

mean beliefs. This confidence will be proportional to the concentrations of the Beta distributions, $\kappa_a \triangleq \alpha_a + \beta_a$ and $\kappa_r \triangleq \alpha_r + \beta_r$. Notice that I explicitly allow also for the risky probabilities to be perceived with less-than-perfect confidence: this will imply that behaviour under risk is itself an outgrowth of cognitive frictions arising in the assessment of known probabilities, as recently modelled in "noisy cognition" accounts of probability distortions under risk (see discussion of related literature below). The inverse of the concentrations can thus be thought of as capturing *perceived ambiguity*. Perceived ambiguity in our setting is thus conceptualized as lack of confidence in the "best guess" about the true underlying probability. This entails that the more ambiguous of the two processes, $p_a(x)$, will have a wider dispersion in beliefs around the true mean. Formally, the concentration for the ambiguous urn is smaller than for the risky urn: $\kappa_a < \kappa_r$. An equivalent statement is that the variance of the ambiguous distribution is wider, i.e. that $\mathbb{V}[\,p_a(x)\,] > \mathbb{V}[\,p_r(x)\,]$, so that a DM is less certain about the ambiguous probability.[3]

Log-odds representation and optimal inference. To show how ambiguity perception translates into choice behaviour, it is computationally convenient to first map the probability distributions into log-odds space. This mapping is insightful inasmuch as it allows me to relate the model predictions to functional forms used in parts of the empirical literature, and to closely related models of decision-making under risk. It also implements a neural paradigm whereby choice quantities are thought to be represented by log-odds coding (Gold and Shadlen, 2001; 2002). This mapping happens without loss of generality: equivalent findings could be derived and characterized directly in probability space.

We can now conceive of $ln\left(\frac{\alpha_r}{\beta_r}\right)$ and $ln\left(\frac{\alpha_a}{\beta_a}\right)$ as unbiased "best guesses" of the probabilities of winning based on the two urns. A rational DM should take these probabilities into account only to the degree that she is confident in their accuracy. Assume the DM has subjective expectations about the distribution of log-odds in

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[3] Concentration and variance capture the same intuition inasmuch as they constitute monotonic mappings of each other. This will be particularly useful when relating our model insights to multiple prior models below. The variance of the belief distribution for the two urns are defined as $\mathbb{V}[\,p_a(x)\,] \triangleq \frac{p\,(1-p)}{\kappa_a+1}$ and $\mathbb{V}[\,p_r(x)\,] \triangleq \frac{p\,(1-p)}{\kappa_r+1}$.

the environment, enshrined in a Bayesian prior $ln\left(\frac{p}{1-p}\right) \sim \mathcal{N}\left(ln\left(\frac{p_0}{1-p_0}\right), \sigma^2\right)$.[4] I make the most parsimonious assumption possible—that the prior is common for risk and ambiguity.[5] Given lack of confidence in the perception of the probabilities entailed by the two urns, the optimal course of action will now dictate to combine the perceived log-odds with the prior expectation. Conditional on the lack of confidence in the estimates, this course of action is optimal inasmuch as it reduces the variance of the estimator, thereby minimizing the mean squared error over many repeated trials (see Bishop, 2006, chapter 3, for a proof of optimality).

The weight attributed to the mean perception relative to the expected log-odds in the prior will depend on the interplay between the confidence in the evidence (the perceived informational content of the urn) and the confidence in the prior expectation. Following the famous characterization of the logit-normal distribution by Atchison and Shen (1980), the *precision* of the perceptions obtains from the parameters characterizing the concentration of the Beta distributions as $\lambda_a = [\digamma'(\alpha_a) + \digamma'(\beta_a)]^{-1}$ and $\lambda_r = [\digamma'(\alpha_r) + \digamma'(\beta_r)]^{-1}$, where $\digamma'$ designates the *Trigamma function*.[6] The DM uses the information in the perceptions and in her expectations, and combines them using weights reflecting the relative precision of the different information sources. This yields a stochastic choice rule predicting the likelihood with which the risky urn is chosen over the ambiguous urn:

$$Pr[p_r(x) \succ p_a(x)] = \Phi\left[\frac{(\rho - \gamma) \times \left[ln\left(\frac{p}{1-p}\right) - ln\left(\frac{p_0}{1-p_0}\right)\right]}{\sqrt{\lambda_r^{-1}\rho^2 + \lambda_a^{-1}\gamma^2}}\right], \tag{1}$$

where Φ is the standard normal CDF. The parameters ρ and γ capture the weight put on the perceived average probability relative to the prior mean, and are de-

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[4] The logit-normal distribution is adopted mostly for expositional clarity, since its conjugacy to the normal likelihood ensures that one can derive a closed form solution for the posterior. Gold and Shadlen (2001) show that the conclusions drawn will be robust to a wide variety of distributional forms, as long as the distributions are single-peaked.

[5] This assumption is logically coherent with the idea that the average probability—the "best guess"—is the same for the risky and ambiguous urn. This assumption is not necessary for the predictions derived here, but to the contrary serves to limit the degrees of freedom in terms of mechanisms through which behaviour can emerge.

[6] Put differently, the *variances* of the subjective belief distributions are defined as from $\sigma_a^2 \triangleq \mathbb{V}\left[ln\left(\frac{p_a(x)}{1-p_a(x)}\right)\right] = \digamma'(\alpha_a) + \digamma'(\beta_a)$ and $\sigma_r^2 \triangleq \mathbb{V}\left[ln\left(\frac{p_r(x)}{1-p_r(x)}\right)\right] = \digamma'(\alpha_r) + \digamma'(\beta_r)$.

termined by the confidence in the perceptions relative to the prior: $\rho \triangleq \frac{\lambda_r}{\lambda_r+\xi}$, and $\gamma \triangleq \frac{\lambda_a}{\lambda_a+\xi}$, where $\xi \triangleq \sigma^{-2}$ is the precision of the prior.

The Ellsberg 2-colour problem. Assume $p = 0.5$, as in the Ellsberg 2-colour problem. The numerator in (1) now simplifies to $-(\rho-\gamma)\, ln\left(\frac{p_0}{1-p_0}\right)$. The definition of ambiguity perception entails that $\gamma < \rho$, so that $\rho-\gamma > 0$. Any prior expectation $ln\left(\frac{p_0}{1-p_0}\right) < 0$ (or equivalently, $p_0 < 0.5$) will thus produce ambiguity aversion, in the sense that it predicts choice proportions of the risky option exceeding 50%. Ambiguity aversion—and the Ellsberg paradox—are thus predicted to occur when the following two conditions are met: a) ambiguity perception in the sense of $\lambda_a < \lambda_r$; and b) a pessimistic prior expectation $ln\left(\frac{p_0}{1-p_0}\right) < 0$. In this sense, risk aversion—here captured by a pessimistic prior expectation—breeds ambiguity aversion when a DM *perceives* the unknown-colour urn as being more ambiguous.

A related intuition whereby risk aversion drives the Ellsberg paradox is presented by Halevy and Feltkamp (2005), where the prediction however emerges from mistakenly perceiving the one-shot task as a *bundle* of choice problems over which bets can be "hedged". Here, the effect derives directly from the second-order uncertainty about the probability of winning in combination with a pessimistic prior expectation, rather than requiring a mis-perception of the choice task.

Generalization to N colours. The model in (1) applies to any probability p, as implemented in multi-colour urns such as used e.g. by Abdellaoui et al. (2011).[7] Assume that $ln\left(\frac{p_0}{1-p_0}\right) = 0$ (i.e. $p_0 = 0.5$). In the presence of ambiguity perception we have $0 < \rho - \gamma < 1$, so that unfavourable odds smaller than 1 (i.e. probabilities $p < 0.5$) will be up-weighed towards 1, resulting in ambiguity seeking for unfavourable odds. The opposite is the case for favourable odds greater than 1, which will be compressed towards 1, so that the less ambiguous process appears more attractive. The immediate implication is that the choices of a given DM

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[7] This includes cases where the number of winning colours is different in the two urns, in which case the numerator in (1) needs to be written as $\rho\, ln\left(\frac{p(R)}{1-p(R)}\right) - \gamma\, ln\left(\frac{p(A)}{1-p(A)}\right) - (\rho-\gamma)\, ln\left(\frac{p_0}{1-p_0}\right)$, where $p(R)$ and $p(A)$ indicate the proportion of winning colours in the risky and ambiguous option, respectively.

will typically not reveal uniform levels of ambiguity aversion (or seeking): in the presence of ambiguity perception, the probability of choosing the risky urn is an increasing function of the proportion of winning colours in the urn.

For a more general prior expectation $ln\left(\frac{p_0}{1-p_0}\right)$, predicted choice patterns will depend on the interaction of the parameters. In particular, stochastic indifference in the sense of 50-50 choice is predicted to obtain at p_0.[8] A pessimistic expectation $ln\left(\frac{p_0}{1-p_0}\right) < 0$ thus yields stochastic indifference at $p_0 < 0.5$. For even odds $ln\left(\frac{p}{1-p}\right) = 0$, a pessimistic common prior $ln\left(\frac{p_0}{1-p_0}\right) < 0$ in combination with ambiguity perception $\kappa_a < \kappa_r$ is thus sufficient to produce the Ellsberg paradox. Probabilities $p < p_0$ will be distorted upwards, and probabilities $p > p_0$ will be distorted downwards in choices, yielding likelihood-insensitivity in ambiguity attitudes.

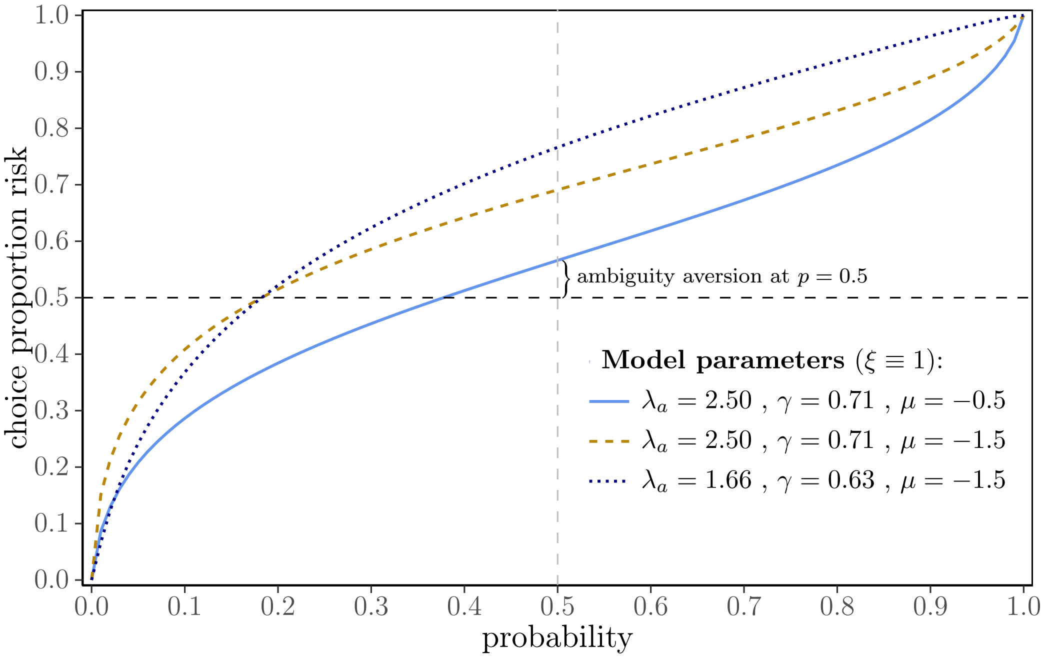

Figure 1: Discriminability between choice options as a function of winning probability

The precision of the log-odds of the less ambiguous process is fixed at $\lambda_r = 10$. The precision of the prior is $\xi = 1$. The solid light blue line indicates ambiguity seeking up to $p = 0.378$, and ambiguity aversion thereafter. Ambiguity aversion for $p = 0.5$ captures the Ellsberg paradox, and is measured by the vertical distance above 0.5. The dashed line showcases the effect of a reduction in the prior mean $ln\left(\frac{p_0}{1-p_0}\right)$. The dotted line further shows what happens when confidence in the log-odds decreases (increased ambiguity perception).

Figure 1 illustrates the mechanism. The discriminability curve given by the solid blue line has $ln\left(\frac{p_0}{1-p_0}\right) = -0.5$ and $\rho - \gamma \approx 0.2$. It displays ambiguity seeking up to

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[8] This is easy to see from equation (1). Setting $p = p_0$ yields $\Phi[0]$, predicting a choice probability of exactly one half over many trials.

a probability $p_0 = 0.378$, and ambiguity aversion thereafter. At $p = 0.5$, it displays ambiguity aversion, thereby capturing the Ellsberg paradox. The dashed brown line has a lower mean of $ln\left(\frac{p_0}{1-p_0}\right) = -1.5$, and hence a lower crossing point at $p_0 = 0.182$. This increases the overall level of ambiguity aversion, although we still witness ambiguity seeking for very unfavourable odds. The dotted line illustrates what happens when confidence in the ambiguous probability decreases, as captured by the smaller value of λ_a. Given the same prior mean $ln\left(\frac{p_0}{1-p_0}\right)$, the crossing point remains the same, but ambiguity aversion at $p = 0.5$ nevertheless increases due to the more pessimistic prior $(\rho-\gamma)\times ln\left(\frac{p_0}{1-p_0}\right)$, which is now driven by the larger value of $\rho - \gamma$. Intuitively, the lower confidence the DM has in the ambiguous process induces her to put more weight on the prior expectation. Given that this prior expectation is pessimistic, the DM chooses to avoid the ambiguous process.

The Ellsberg 3-colour problem. Take an urn with 90 balls: 30 are red, and the remaining 60 either green or blue. When asked to bet on either red or blue to win a fixed prize x, a majority of subjects typically prefer betting on the red ball. When asked to bet on red or green versus blue or green for the same prize, a majority of subjects typically prefer blue or green. The significance of this example stems from the fact that it violates the sure thing principle: an identical event being added to both options in the original bet causes the choice to flip. The intuition is that the probability of blue may be perceived as more ambiguous than the probability of red, since we do not know the exact proportion of blue balls in the bag. Once the bet on blue is mixed with the bet on green, however, we do know that proportion: the mix between red and green has thus become the more ambiguous option.

The model predictions are straightforward. In the choice between red and blue, blue is perceived as more ambiguous. In the choice between red or blue and green or blue, however, red and blue is more ambiguous. Given this ambiguity perception and a pessimistic prior, the DM will choose red in the first choice option, but green and blue in the second.

Gains versus losses. Empirical choice frequencies in the types of tasks discussed above are often found to flip when the prize is exchanged for a loss (Baillon and Bleichrodt, 2015; Trautmann and van de Kuilen, 2015). Following the noisy cognition literature, I here assume that gains and losses are perfectly discriminated (Khaw, Li and Woodford, 2021; Vieider, 2024). The expression in square brackets in equation (1) then ought to be multiplied by -1, yielding a prediction of ambiguity aversion for small log-odds (which now ought to be considered *favourable*), and of ambiguity-seeking for larger log-odds.[9]

2.2 Learning and probabilistic updating

The model introduced above predicts rich behaviour under ambiguity based on a highly tractable Bayesian statistical framework. A consequence of this is that it naturally integrates learning. One can conceptualize learning about the ambiguous process as a series of draws of successes s and failures $1-s$, which can be added to the original perceptual parameters α_a and β_a encoding the ambiguous probability. As well as potentially adjusting the perceived average probability, such learning will increase the confidence a DM has in the probability assessment by increasing the precision $\lambda_a = \sum_i \left[F'(\alpha_a + s_i) + F'(\beta_a + (1-s_i))\right]^{-1}$, which is an increasing function of draws i. This is predicted to yield an increase in $\gamma \triangleq \frac{\lambda_a}{\lambda_a+\xi}$, and hence a reduction in the likelihood-dependence of ambiguity attitudes, and an increase in ambiguity tolerance.

These predictions are consistent with the existing empirical literature. Heath and Tversky (1991) famously showed how ‘competence’, or familiarity with a source of uncertainty, could lead to preferring such ambiguous sources over known risk. This is predicted to happen based on the modelling setup here presented if $\gamma > \rho$ for familiar sources. Ert and Trautmann (2014) show that upon sampling DMs reverse their choices in favour of the ambiguous option, and that choices of the supposedly more ambiguous option *increase* in the probability of winning—a finding that is highly consistent with the mechanism modelled here. In Oprea and Vieider (2024),

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[9] This flipping in attitudes does not need to be perfect. For instance, presenting losses versus gains may trigger different levels of attention, which could in turn influence the confidence parameters ρ and γ. See Bouchouicha, Li and Vieider (2025) for a discussion of this point in the context of mixed gain-loss choices.

we show that sampling affects behaviour not only for ambiguous options, but also for risky options, supporting the idea that even fully described risks may be encoded with less than perfect confidence.[10]

3 Discussion and Conclusion

I have presented a model in which ambiguity attitudes result from conceptualizing probabilities based on the tools of Bayesian statistics. While the process is fully Bayesian—and can thus be considered normative—the model diverges significantly from the Savagian Bayesian updating of point beliefs (see Gilboa and Marinacci, 2016, for a discussion of this approach and its descriptive limitations). The approach is highly principled, tractable, and yields predictions that are consistent with some of the key findings in the empirical literature.

Relation to multiple prior models. The definition of ambiguity perception as "lack of confidence" we use is consistent with definitions of sets of priors as capturing "revealed ambiguity perception" in Ghirardato et al. (2004) and with the notion of "ambiguous beliefs" in Klibanoff et al. (2005). In this sense, the notion of uncertainty about the true probability we take as our starting point coincides with the notion of "set of priors" (Gilboa and Schmeidler, 1989), even though in our Bayesian context it ought to be technically described as uncertainty in the *likelihood* function. The basic intuition underlying the model is thus aligned with the intuition about model uncertainty arising from multiple prior models.

The model can thus be seen as providing an empirically tractable continuous approximation to sets of priors held by a DM. An important difference is that the benchmark under risk is not EU maximization as in maxmin, α-maxmin and the smooth model. Rather, EU maximization is a special case which obtains when described probabilities ("risk") are perceived without any uncertainty. In that case, the Beta distribution converges to a Dirac-delta that has all its probability mass

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[10] Learning may well depend on the presentation format of new information, with such information itself being perceived as noisy: developing such a framework is left for future research.

associated to the objective probability, $\mathcal{B}e(\alpha_r, \beta_r) \xrightarrow[\alpha_r, \beta_r \to \infty]{} \delta(p_r - p)$, so that $\rho \to 1$. In general, however, the model allows for $\alpha_r + \beta_r < \infty$ and hence for $\rho < 1$.

Ambiguity perception and source theory. Source theory models ambiguity attitudes using a two-stage model where probabilistic beliefs take the form of probability point estimates like in EU. Ellsberg-like behaviour is then captured by subjective distortions of those beliefs. Let $w_r(p)$ be the source function for risk, and $w_a(p)$ the source function for ambiguity. A DM will prefer to bet on the risky urn whenever $w_r(\widehat{p}_r) > w_a(\widehat{p}_a)$ (Abdellaoui et al., 2011). The typical finding in the empirical literature is that the discrepancy between w_r and w_a increases in the probability p, a phenomenon that is called *ambiguity-induced likelihood-insensitivity*. This discrepancy is captured by a matching probability—the probability under risk that makes the DM indifferent between taking the risky or the ambiguous urn—defined as $p_m = w_r^{-1} \circ w_a(\widehat{p}_a)$ (Baillon et al., 2025).

It is straightforward to show that equation (4) provides cognitive micro-foundations for such a setup. Define an ambiguity-weighting function $w \triangleq w_r^{-1} \circ w_a$. To obtain a mapping between this function and the predictions derived above, we use the popular function proposed by Goldstein and Einhorn (1987), which has the following linear in log-odds (LLO) form, characterized by Gonzalez and Wu (1999):

$$ln\left(\frac{w(p)}{1 - w(p)}\right) = \eta\, ln\left(\frac{p}{1-p}\right) + ln(\delta), \tag{2}$$

where $\eta < 1$ captures likelihood-insensitivity in ambiguity attitudes, and δ captures optimism, i.e. the opposite of ambiguity aversion. This functional form obtains from the numerator in (1) by defining $\eta \triangleq \rho - \gamma$ and $\delta \triangleq exp\left[(\gamma - \rho)\, ln\left(\frac{p_0}{1-p_0}\right)\right]$. The LLO weighting function for risk will thus be $\rho\, ln\left(\frac{p}{1-p}\right) + ln(\delta_r)$, where $\delta_r = exp[(1-\rho)\, ln\left(\frac{p_0}{1-p_0}\right)]$; and the LLO function for ambiguity will be $\gamma\, ln\left(\frac{p}{1-p}\right) + ln(\delta_a)$, where $\delta_a = exp[(1-\gamma)\, ln\left(\frac{p_0}{1-p_0}\right)]$.

Other than source theory, our model predicts choice to be inherently stochastic. It can thus explain the high levels of noise that are typically found in choices under ambiguity (L'Haridon, Vieider, Aycinena, Bandur, Belianin, Cingl, Kothiyal and

Martinsson, 2018). Choices under risk are seen as an outgrowth of cognitive frictions in in the coding of objectively described probabilities, thereby endogenizing parameters governing risk-taking (Khaw et al., 2023; Vieider, 2024). In Oprea and Vieider (2024), we test this proposition by forcing experimental subjects to draw representative samples from fully described choice options. While in the absence of such samples risk-taking declines systematically in the probability of winning a prize, consistently with inverse-S probability weighting, forced sampling eliminates likelihood-dependence and results in “standard” behaviour consistent with EU maximization. This showcases the endogenous nature of probability distortions under risk that constitutes the benchmark case of the model presented above.

Concluding remarks. The formal link of the model I have presented here to models of noisy cognition holds the promise of a unifying framework under which to characterize decision-making across a variety of contexts. The same process model has indeed been shown to account for small-stake risk aversion (Khaw et al., 2021), likelihood-dependent risk-taking (Zhang et al., 2020; Khaw et al., 2023; Vieider, 2024), the description-experience gap (Oprea and Vieider, 2024), and all the major empirical regularities in delay-discounting (Vieider, 2023).

At the same time, many interpretational questions remain open. The model I presented here could be construed as the dual model of the one over mixed gain-loss lotteries proposed by Khaw et al. (2021). Testing that model, Bouchouicha et al. (2025) conclude that increased sensitivity to losses relative to gains is the main driver of behaviour. While the evidence weight for risk versus ambiguity plays a similar role in this model, the relative informational content of the two choice options is clearly different in these two contexts (i.e., there is objectively missing information in the description of the ambiguous urn; Frisch and Baron, 1988). This raises questions about how internal cognitive frictions may interact with informational frictions arising exogenously—a priority topic for future research.

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